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Natuurkunde. — L. S. ORNSTEIN, namens J. F. SCHOUTEN: *De rol van electrische, photochemische en diffusie verschijnselen bij het zien.*

(Communicated at the meeting of April 24, 1937).

Door den heer J. F. SCHOUTEN zijn in het Physisch Laboratorium te Utrecht onderzoekingen verricht, die van belang kunnen zijn voor de verklaring van de processen, die zich bij het tot stand komen van den gezichtsindruk op het netvlies afspelen.

Door SCHOUTEN werd een methode gevonden, die naar ons later bleek, in 1934 door W. D. WRIGHT was gebruikt. De vraag, die wij ons stelden, was den invloed na te gaan, dien belichting van deelen van het oog op de gevoeligheid van andere plaatsen uitoefent. Om de gevoeligheidsverandering of dus verblinding quantitatief te meten, werd de lichtindruk, dien het in een willekeurig punt bestraald oog van een verlicht vlakje krijgt, vergeleken met dien, welke het tweede, niet bestraalde, oog van een gelijk vlakje vertoont. Is de gevoeligheid van het bestraalde oog verminderd, dan moet het vlakje, dat het beschouwt sterker belicht worden, dan het vlakje, dat aan het onverblinde oog wordt aangeboden, of wat op hetzelfde neerkomt, dit laatste minder sterk belicht worden om de twee beschouwde vlakjes gelijk te zien. De methode is mogelijk, omdat het onbelichte oog niet of weinig verandert. Bij deze methode neemt men dus in compensatie waar. Bepaald kan dus de breuk waarmede het licht op het vlakje, dat het onverblinde oog ziet, verzwakt moet worden om even helder te worden waargenomen als het door het verblinde oog geziene vlakje.

Bij de eerste waarnemingen, die gedaan werden, werd het verblinde oog op verschillende hoekafstanden van de fovea met dezelfde intensiteit bestraald en de verandering r als functie van den tijd waargenomen. De gevoeligheid bleek zeer snel te dalen en dan constant te worden. Staakt men de belichting, zoo springt r een stuk naar boven en verloopt vervolgens langzaam, totdat zij uiteindelijk de waarde 1 weder bereikt. De gevoeligheidsvermindering is des te grooter, naarmate het verblindende lichtpunt dichter bij de fovea ligt, terwijl het herstel des te langzamer is naarmate deze afstand kleiner is. Bij langdurige verblinding blijft de gevoeligheid na de eerste adaptatie (α) constant, de tweede adaptatie (β) treedt dan zonder sprong op. Het beschreven verschijnsel is nu als functie van de plaats ten opzichte van de fovea (in hoekmaat) en de

intensiteit van het licht onderzocht. Er blijkt dan, dat $F\ (=\frac{1}{r}-1)$ evenredig is met de intensiteit en omgekeerd evenredig is met den afstand tot de fovea. Verder blijkt, dat de α-adaptatie onafhankelijk van de kleur is. Wordt namelijk het oog met blauw licht verblind en aan beide oogen een rood en blauw verlicht vlakje voorgelegd, dan blijkt de

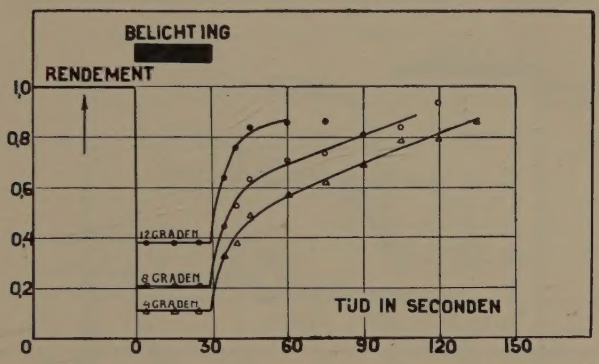


Fig. 1. *Het rendement als functie van den tijd tijdens en na de verblinding voor verschillende hoeken.*

Het rendement r zakt bij den aanvang van de belichting momentaan tot een lagere en constant blijvende waarde. Na afloop van de belichting vindt een aanvankelijk snel en later langzaam herstel plaats. Voor kleinere hoeken tusschen blindingslichtbron en meetvlakje liggen de curven lager.

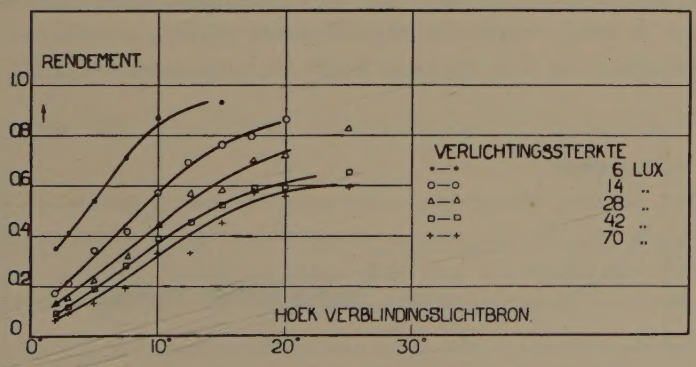


Fig. 2. *Het rendement tijdens de belichting als functie van den hoek voor verschillende verlichtingssterkten op het oog.*

De belichting van een bepaald punt van het netvlies brengt daaromheen als het ware een kuil in de ooggevoeligheid te weeg.

gevoeligheidsvermindering voor beide kleuren (rood en blauw) bij de blauwbestraling gelijk te zijn. Wordt dan de verblindende blauwverlichting gestaakt, dan neemt men voor het roode object den adaptatiesprong waar, voor het blauwe niet, terwijl beide vervolgens een tweede (β) adaptatie vertoonen.

Het lag voor de hand te pogen na te gaan of de α-adaptatie van den tijd afhangt. Daartoe was het noodig zeer korte verblindende flitsen te geven en op verschillende oogenblikken na den flits de gevoeligheidsvergelijking met de beide oogen uit te voeren. De heer SCHOUTEN deed dit op de volgende wijze. In een wiel (fietswiel) werden twee spleten aangebracht, waarvan zoowel de breedte als de relatieve plaats veranderd

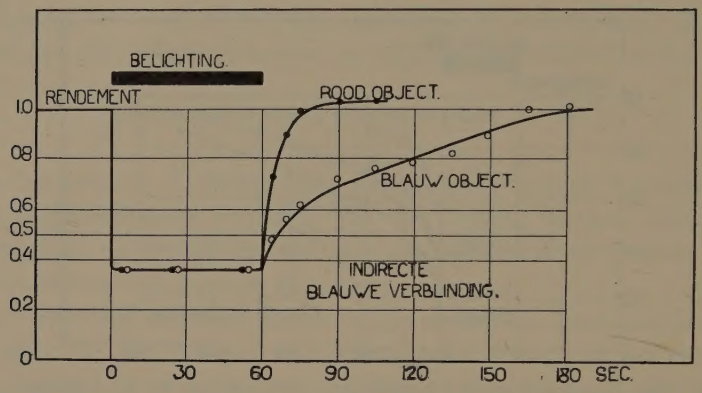


Fig. 3. *Het rendement als functie van den tijd tijdens en na de verblinding voor een rood en een blauw object bij indirecte blauwe verblinding.*
Het rendement is voor beide kleuren tijdens de belichting vrijwel gelijk, daarna echter heeft voor rood een veel sterkere sprong plaats dan voor blauw.

konden worden, de eene dicht bij, de andere verder van de as. De eene spleet laat het verblindende licht toe; een korten tijd later kunnen door de andere met de beide oogen de twee verlichte vlakjes beschouwd worden. De omwenteling van het wiel geschiedt in 1 seconde. De duur der ver-

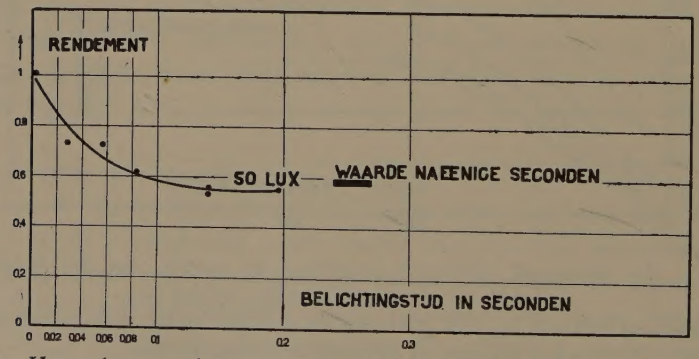


Fig. 4. *Het verloop van het rendement gedurende den aanvang van de belichting. Meting met de spletenschijf.*
Het rendement zakt binnen 0.2 seconden tot een constante waarde welke gelijk is aan die, welke bij lange belichtingen (meer dan 2 seconden) gevonden wordt.

blinding wordt gevarieerd door de breedte der spleet. Het bleek nu, dat de gevoeligheidsverandering, onmiddellijk na den flits waargenomen,

continu verloopt en dat als functie van den duur der verblinding in 0,04 secunde de helft der uiteindelijke gevoeligheid bereikt werd, die na ongeveer 0,2 sec. belichting tot stand komt en dezelfde waarde als de gevoeligheidsverandering bij stationnaire waarneming bezit.

Op deze wijze is nu de grootheid F als functie van de intensiteit en van den hoek van het verblinde punt met de fovea bepaald. Deze hoekafhankelijkheid vertoont een merkwaardige analogie met electrische

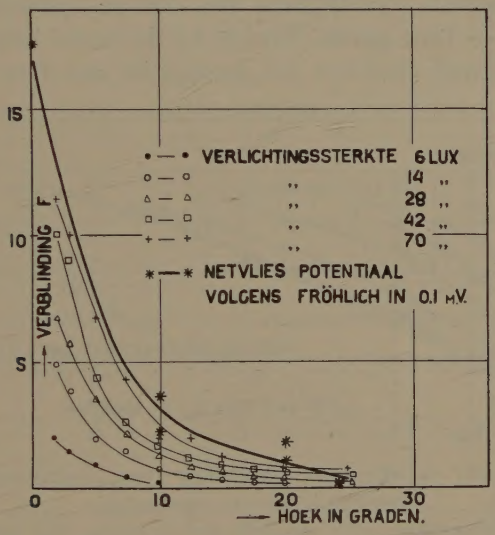


Fig. 5. *Analogie tusschen verblinding en netvliespotentiaal als functie van den hoek tusschen verblindingslichtbron en meetplaats.*

De curven voor de verblinding $F = \frac{1}{r} - 1$ vertoonen een zelfde hoekafhankelijkheid als die welke door FRÖHLICH voor de netvliespotentiaal gevonden werden.

proeven, die FRÖHLICH aan inktvischoogen heeft gedaan, waarbij de invloed der bestraling van verschillende punten van het oog door de grootte van den actiestroom werd gemeten met een electrode op verschillende punten van het netvlies. De kromme van FRÖHLICH past geheel in het stel der door SCHOUTEN waargenomen curven. De verklaring, die FRÖHLICH geeft, door op de verstrooiïng van het licht in het oog te wijzen, is onjuist. Dit wekt het vermoeden, dat het verschijnsel der α-adaptatie van electrischen aard is — dit was reeds te verwachten bij een dergelijk snel verschijnsel; de genoemde overeenstemming bevestigt dit verder, terwijl ook het feit, dat, indien een stroom het oog in één richting doorloopt, een lichtindruk ontstaat, bij doorloopen in de tegengestelde richting echter een gevoeligheidsvermindering ontstaat, verder in deze richting wijst. Wij kunnen ons dientengevolge voorstellen, dat het eerste proces in het oog een photo-electrische electronenemissie is, welke de potentiaal van een electrische dubbellaag verandert en zich in de omgeving der bestraalde

plaats als potentiaalverandering en daarmede als gevoeligheidsvermindering doet voelen.

Het feit, dat de α-adaptatie bij langere bestraling van karakter verandert, (het wegblijven van den sprong) bij overigens gelijke gevoeligheidsvermindering, toont, dat naast het eerste proces een tweede zich af gaat spelen, waardoor de potentiaalverschillen vereffend worden. Er blijven waarschijnlijk photochemische veranderingen optreden.

Omtrent deze verschijnselen kan een nadere studie der β-adaptatie — van het herstel der gevoeligheid dus, dat na ophouden van het verblinden optreedt — licht geven. Terwijl bij de eerste besproken adaptatie geen kleurverandering (behalve een geringe bij den flits) optreedt, blijkt

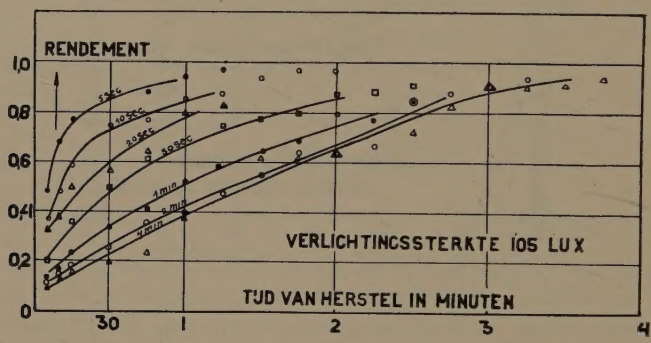


Fig. 6. *Het herstel na afloop der verblinding voor verschillende verlichtingssterkten bij directe verblinding.*
Voor korte belichtingstijden vindt een snelle sprong plaats, voor lange daarentegen neemt het rendement geleidelijk tot de eindwaarde 1 toe.

de β-adaptatie kleurafhankelijk te zijn. Is bijvoorbeeld met groen licht verblind, zoo blijkt de groengevoeligheid in en om de belichte plaats te zijn afgenomen; wordt dan met wit licht bestraald, zoo wordt het roodachtig waargenomen, terwijl de compensatiewaarneming met gekleurd licht moet geschieden. Zoowel voor directe adaptatie (kijken in de lichtbron), welke reeds door WRIGHT werd onderzocht, als bij indirecte verblinding blijkt, dat er een langzaam herstel der gevoeligheid bestaat. Indien men voor verschillende tijden der verblindende bestraling de gevoeligheidstijdkrommen uitzet, daarbij als nul van den tijd kiezende het oogenblik, dat de verlichting begon, zoo vindt men een schaar, waarin ordinaten op gelijke tijden met de belichtingstijden der verblinding evenredig zijn — het maakt dus den indruk, of er evenredig met dezen tijd een photochemische afbraak van een stof heeft plaats gehad, welke zich slechts langzaam herstelt.

Het feit, dat hier een langzaam proces optreedt, hetwelk zich vanaf de bestraalde plaats voortplant, zou den indruk van een diffusieverschijnsel kunnen wekken. Toch schijnt het onwaarschijnlijk, dat de wederzijdsche beïnvloeding aan diffusie der photochemische afbraakproducten is toe te schrijven. Het feit, dat nabeelden hun vorm behouden tijdens de herstel-

reactie, wijst hier reeds op. Proeven om de diffusie aan te toonen door op verschillende afstanden van de fovea met verschillende kleuren te verblinden, toonden geen effect van voortplanting, gelijk dit bij diffusie te verwachten is.

De wederzijdsche beïnvloeding moet dus wel aan verbinding der lichtgevoelige elementen worden toegeschreven, die potentiaalverschillen kan voortplanten, terwijl de vorming van stoffen in de bestraalde elementen

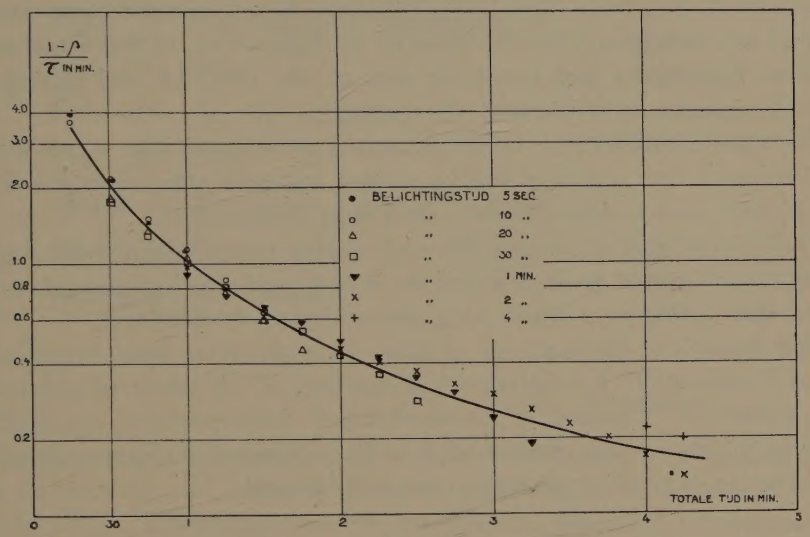


Fig. 7. *Het eigenaardige verband tusschen de herstelcurven voor verschillende belichtingstijden bij directe verblinding.*

Wordt de grootheid $\frac{1-r}{\tau}$ waarin τ den belichtingstijd voorstelt, uitgezet tegen den tijd verloopen sinds het *begin* van de belichting, dan blijken de punten voor verschillende belichtingstijden op één curve te liggen.

concentratie-potentiaalverschillen naast de photo-electrische zouden kunnen veroorzaken. Het langzame herstel zou dan te wijten zijn aan een langzaam verloopende chemische reactie van de afbraakproducten der photochemische reactie, die om twee redenen geen groote snelheid zou hebben, ten eerste omdat het een oppervlaktereactie zou zijn, ten tweede omdat de in de buurt der geleidingen weggediffundeerde afbraakproducten slechts langzaam zouden terugdiffundeeren. Een andere mogelijkheid zou zijn, dat de gevormde producten door de dubbellaag buiten de kegels zouden treden over afstanden geringer dan de kegeltjes zelf en dat zij na het ophouden der belichting, waarbij de oorzaak der potentiaalverandering waardoor zij konden uittreden, ophoudt, weer naar de kegeltjes zouden terugkeeren en daar het nabeeld en in de omgevende kegeltjes het herstel der dubbellaagpotentiaal door het vereffenen der concentratieverschillen zouden veroorzaken.

Het moge zijn, dat deze beschouwingen een hypothetisch karakter hebben — iets in deze richting suggereert het typische verschijnsel der snelle α-, blijkbaar photo-electrische en der langzame β-adaptatie toch zeker.

Summary.

Stimulation of a part of the retina by light produces a sudden fall in the sensitivity of the fovea. This fall is measured by a binocular compensation method as a function of the intensity of the lightsource, of the visual angle between lightsource and testobject and of the time, as well during the time of exposure as of the time of recovery.

The chief characteristics of the occurring phenomena are: a sudden fall ($\backsim$ 0.1 sec.) to a *constant* level, a sudden recovery after short exposures ($<$ 10 sec.) and a slow recovery after long exposures.

The phenomenon of sudden fall and sudden rise is called α-adaptation. α-adaptation should be interpreted as a photo-electrical phenomenon, the retina thus acting as a whole as a photo-electric double-layer.

The secondary phenomenon of slow recovery after long exposures is called β-adaptation. β-adaptation is a function of the colour of lightsource and testobject. Theories of photo-chemical decomposition can scarcely account for the strong relation of β- with α-adaptation. Thus possibilities of diffusion over short distances are put forward.

Physics. — *The Gravity Expedition of Hr. Ms. Submarine O 16 in the North Atlantic, January 11 — March 16, 1937.* By F. A. VENING MEINESZ.

(Communicated at the meeting of April 24, 1937).

The plans of the Netherlands Navy for a cruise of Hr. Ms. Submarine O 16 in the Atlantic offered an attractive opportunity for important gravity work and so the Netherlands Geodetic Commission addressed a request to the Minister of Defence, H. E. Dr. H. COLIJN, for allowing the author to go along and to make gravity observations during the trip. The request was granted and the authorities of the Navy have kindly consented to adjust the programme of the voyage to the scientific desiderata. The result was an expedition of great importance and the author wishes to express his sincere acknowledgements for this renewed instance of extensive cooperation after so many other notable instances in the past years.

January 11 Hr. Ms. O 16 left the Naval Base of Den Helder and the following programme was accomplished:

1937	Ports	Observation Stations
Jan. 11	Dep. Den Helder	17
Jan. 20	Arr. Horta (Azores)	1
Jan. 25	Dep. „	24
Feb. 5	Arr. Hamilton (Bermudas)	1
Feb. 10	Dep. „	8
Feb. 15	Arr. Washington	1
Feb. 24	Dep. „	28
March 7	Arr. Ponta Delgada (Azores)	1
March 8	Dep. „ „	11
March 12	Arr. Lisbon	1
	Total	93 stations

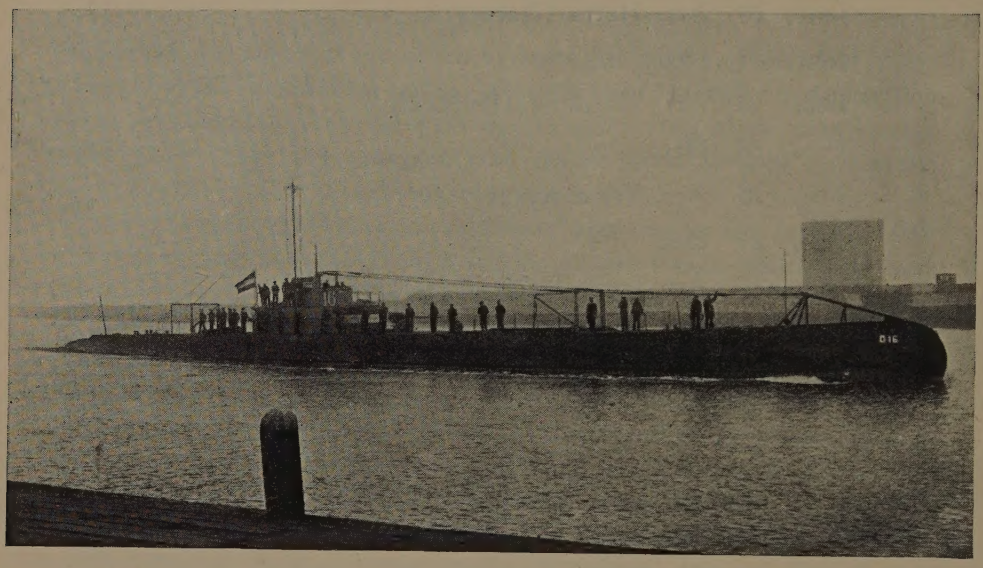
The list mentions the number of stations where observations have been made in each leg and, besides, the observations made in the ports; in the ports of Horta, Washington and Ponta Delgada, these were repetitions of former observations and so the total number of new gravity stations is 90. The stations are plotted on the map and the route is indicated by a full-drawn line. The scientific work was ended in Lisbon as the ship got special instructons there in connection with the Spanish Civil War. The author left for Holland in an ordinary ship and arrived there on March 25.

The staff of Hr. Ms. O 16 consisted of

Lieutenant Commander C. J. W. VAN WANING, Captain,
Lieutenant J. F. VAN DULM, Executive Officer,
Lieutenant K. C. J. LUGTEN, in command of the engine-room,
Lieutenant H. A. W. GOOSSENS, Navigator,
Lieutenant A. OHR, second in command of the engine-room,
Lieutenant A. H. W. VON FREYTAG DRABBE.

The author feels indebted to all of them for their helpful assistance during the trip and for their good friendship, which will make him remember this trip with the kindest feelings. He is especially indebted to Captain VAN WANING for the great number of dives, which allowed the entire execution of the scientific programme, notwithstanding the great difficulties and trouble often adherent to the dives because of the exceptionally rough sea. He wishes also to mention the trouble which Lieut. GOOSSENS has given himself for the determining of the positions for which he showed a remark-

able ability; although the sea was very rough, the positions were mostly accurate up to two miles and often even to one mile or less. He wishes lastly to mention the assistance given by the petty officers and the crew, and especially the names of Sergt. Major SCHIEVEEN who helped him along with small reparations of his instruments, and of Sergt. ZWEISTRA, the



Chief Radioman, and his assistants who tuned in for the time-signals and who did the echo-sounding.

The voyage was an exceptionally rough one. After one calm day after leaving Den Helder, the wind rose to gale force and did not diminish till many weeks after. One gale followed the other and the sea rose to towering wildness. After Horta three times hurricane force was reached. It was all against us and so the speed of the ship was reduced by several knots, from time to time even by five or six knots. The lower deck was of course quite inaccessible but mostly even the remaining on the conning-tower was an extremely wet proposition; many days occurred that heavy masses of water reached up as far as that. Inside the ship life was not simple and it is scarcely necessary to say that the rolling and pitching was terrible. Under these conditions the nights were restless and the life on board was indeed strenuous. Diving was difficult and so a tribute may be paid to the Captain for continuing to do so. Once down, it was a relief for everybody and in order to give a short spell of rest and to ensure a meal under less-fantastic circumstances than usual, the Captain decided a few times to stay down for some hours instead of rising after the normal time of one hour for the observation. This opportunity was always used for making a second observation at the end of the dive and so the map shows a few instances of pairs of stations at small distances of 10 to 20 miles from each other. The second station cannot of course be considered as an important

new value, but it is a check on the reliability of the observations and also on the reliability of the corrections for topography, because the effect of the topography may be entirely different for the two stations. Besides, valuable indication is obtained about the rate of change of gravity in the sense of the route.

The trip from the Bermudas to Washington was a smooth one, but soon after leaving Washington the rough weather came back with renewed force. The wind and waves, however, came from behind and so the speed was less impeded, but from time to time solid waves swept over the conning-tower and drenched the officers of the watch and the man at the wheel up to their necks. The last days before Lisbon were quiet again.

The gravity observations were made with the old multiple pendulum apparatus used during former expeditions. Three chronometers have been taken along for the checking of the rates; besides the two Nardin chronometers of the Netherlands Geodetic Commission one chronometer was kindly lent by the Observatory of Leiden; the author wishes to express his acknowledgements for this valuable support to its Director, Prof. Dr. E. HERTZSPRUNG. Two chronometers kept sidereal time and one mean time; all three had electric break-circuit arrangements. The first two ones were switched on in the photographic recording of the pendulum apparatus and all three chronometers were compared before and after the pendulum observation by means of a small recording apparatus (built by Boulitte, Paris) of which the writing pens were activated by electric currents of

$2\frac{1}{2}$ milliamp. The difference of the rates of the two sidereal time chronometers deduced from the pendulum records, may thus be checked by the difference of the rates determined by the Boulitte recording instrument. The rate-check of the third chronometer can only be used by means of this latter instrument.

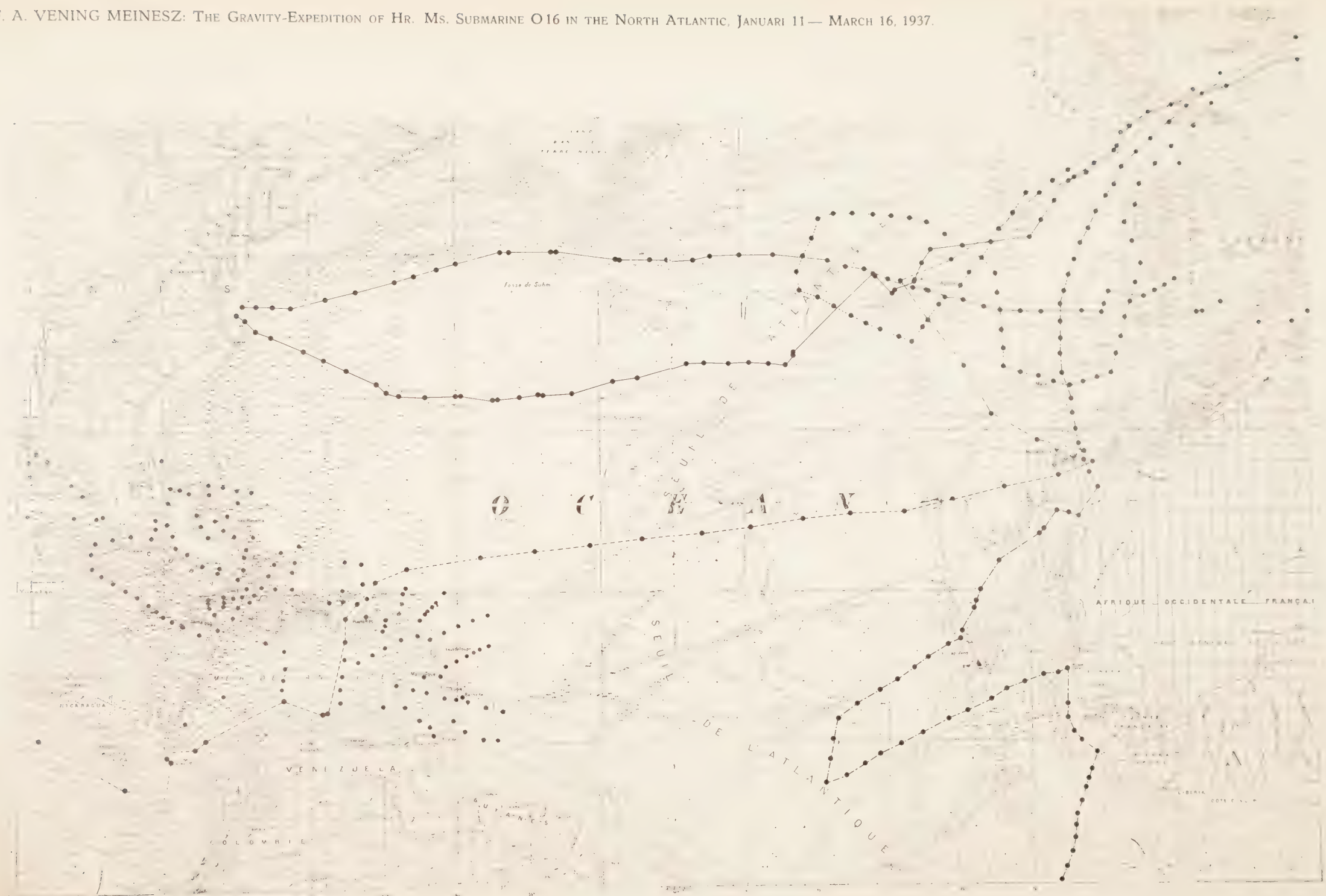
The submarine was provided with a Fessenden sound emitting and receiving installation and this was used for echo-sounding. The time-interval between sound and echo was determined by means of an accurate stop-watch, of which the dial indicates the fiftieth of a second and which thus allows without difficulty to read the hundredth of a second. The radioman pressed down at the same time with one hand the handle of the sound emitter and with the other one the knob of the watch; the coincidence of both actions was usually accurate up to one or two hundredths of a second. The time of the echo was read on the dial without stopping the pointer. In this way no time-lag entered into the determination of the time-interval.

Although the sound-emitter and the receiver were not unfavourably located, it has not been possible to make soundings during the trip at the surface of the water. The energy of emission was rather small and the wave-disturbance was too big. So the sounding has been only possible during submergence. In normal circumstances depths were then taken every quarter of an hour but when irregularities occurred they were taken at smaller intervals; the speed of the ship during submergence was 3.3 sea-miles per hour.

For the check of the chronometer-rates the radio time signals of the rythmic type of the French station F. Y. L. at 8 h. 01 and 20 h. 01 Greenwich time have been used; save on one or two occasions no trouble has been experienced in receiving them. The comparison with the chronometers was made according to the method of Cooke; the break-circuit of one of the chronometers was put in series with the telephone.

As the map shows, the stations of this expedition are a valuable addition to what was already known from previous work. Together a fairly complete survey has been obtained of a large part of the North Atlantic and a more detailed net of stations was got for the eastern part. The route of this trip is indicated by a full-drawn line and the routes of former expeditions — of Hr. Ms. K XIII, O XIII and K XVIII — by dotted lines; the few stations near Europe obtained with Hr. Ms. K II and K XI have been added. The stations made by three expeditions of the U. S. Navy in the West Indies and adjoining waters by the U. S. S. S 21, S 48 and the „Barracuda" — have likewise been put on the map.

Only a few observations have been computed at this moment and so no conclusions can yet been drawn, but a short review may be given of the problems about which the author hopes that results may be got. The first and most general one is the question whether the wide and deep basins of the North Atlantic all show the fields of positive anomalies, which up to now have been found in several of the Atlantic basins. For the great basin between the U. S. and the Mid Atlantic ridge there was some doubt in this regard, because the only available stations were a few in the crossing of the K XIII from the Canary Islands to Porto Rico, of which the western


Fosse de Suhm
SEUIL DE L'ATLANTIQUE
SEUIL DE L'ATLANTIQUE
VENEZUELA
COLOMBIE
AFRIQUE OCCIDENTALE FRANÇAI

ones are too near the great Nares Deep for giving undisturbed values for the basin; the few remaining ones did not give clear indications. The new trip has given a great number of stations in two important crossings and so it may be assumed that a definite conclusion will be possible. Three of the stations east of the Bermudas have already been computed provisionally and they all show positive anomalies.

This problem is an important one because it appears quite unlikely that the rigid crust of the Earth could be strong enough to bear the mass-loads corresponding to these extensive anomaly fields by its own strength; it therefore seems unavoidable to look for the cause in disturbances in the deeper plastic layers and this would lead to the assumption of the great current systems in the Earth, which the author had already the occasion to enlarge on in previous publications. The importance of this problem need not again be stressed.

The author wishes to avail himself of this opportunity for elucidating a point which has already been expressed by him on former occasions but which has been misunderstood by more than one author who has written about these subjects. In introducing the assumption of a rigid crust floating on a plastic substratum, it is not necessary to assume a sudden transition between both layers; a gradual change of physical properties would leave the essential conclusions inherent to this supposition unaltered. Neither is it necessary to assume that the transition coincides with the chemical transition, usually indicated by the denomination sial-sima, or with the transition crystalline-amorphous. Nor is it necessary to assume that the rigid crust is a homogeneous layer; it may consist of more than one different layers. The only assumption that is needed is that from the surface downwards, in some way or another, the elastic limit diminishes and that the specific gravity and the plasticity increase. Starting from these general assumptions, which seem likely from all view-points, we may draw the conclusions used in former publications [1]) about the crust floating on the substratum and — under special conditions — buckling downwards under lateral compression.

A second problem about which the new expedition may give new figures is the question how the transition occurs of the field of positive anomalies in the ocean-basins to the field of normal values beside them; during previous expeditions a rather sudden transition has usually been found. The knowledge of the character of the transition may be helpful for finding the right interpretation of these anomaly-fields.

Beside the general problems, several more local problems may be furthered by the material of the expedition. We may first mention the question whether the gravity field west of the Iberian Peninsula points to the continuation of the folded belts of the European continent into the

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[1]) See e.g. Gravity Expeditions at Sea, 1923—1932, Vol. II, Chapt. I, P. III, § 5 a. f., Chapt. V, § 2 a. f

Atlantic and, if so, in which way. Do they continue towards the Azores or do they turn back in the way STILLE supposes [2])? The fact that former expeditions have shown the gravity anomalies in this area to be smaller than the anomalies found in the folding-belts in the East and West Indies is not surprising; it does not necessarily point to older foldings of which the anomalies have already partly disappeared. We may assume the rigid crust under the oceans to be considerably thinner than in the continents. It could, therefore, be easily explained that the irregularities of mass brought about by the lateral compression of this thinner crust, are smaller. If we assume e.g. a concentration of lighter crustal material at the lower side of the crust, caused by a downward buckling of the crust, this mass-concentration is not only proportional to the amount of horizontal compression but also to the thickness of the crust.

The fact, however, that the gravity anomalies are smaller makes more gravity figures necessary before we can conclude about a continuous belt of anomalies of the kind we are looking for. We cannot connect irregularities in gravity profiles at great distances of each other as we could do in the East Indies, where the irregularities are so much more striking. The old gravity material was not sufficient to come to safe conclusions, but the author hopes that the addition of the great number of new stations will allow to settle the question. Definite conclusions will, of course, be important for the problem of WEGENER's hypothesis; the presence of folding-belts would indicate a rigid crust below the Atlantic and this does not seem easily compatible with this hypothesis.

The expedition has given two new gravity profiles over the Azores; one from W.N.W. to E.S.E. over the islands and one perpendicular to this direction. Combined with the old profiles in this neighbourhood, given in 1932 by the expedition of Hr. Ms. O13, new insight in the problems of the way of formation of this archipelago may be obtained. Is the origin purely volcanic on about east-west running fault-lines, or is a tectonic phenomenon at the base of it, as the earthquake-centres seem to indicate?

New insight may likewise result for the Bermudas. Will the gravity profile over this archipelago indicate isostatic equilibrium, or will deviations be found pointing to a tendency for vertical movements for the readjustment of the equilibrium?

For these and other problems, the results of the expedition will have to be carefully studied after their being subjected to different methods of isostatic reduction. Up to now, the assumption of regional isostatic compensation could only be applied for one figure for the degree of regionality and for a limited range of assumptions for the depth of compensation. The Netherlands Geodetic Commission has now taken up the computation of tables for a wider field of application of this method. The tables will cover

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[2]) HANS STILLE, „Zur Frage der Transatlantischen Faltenverbindungen", Akad. d. Wiss., Berlin, 1934, XI.

a wide range for the degree of regionality, spreading the compensation over areas with radii of about 0, 50, 100, 150 or 200 km., and they will allow many kinds of suppositions for the depth of the compensation between 0 and 60 km. They will be available in the course of this year.

The computation of the observations of the expedition have been started at once after the return in Holland and so the results for the gravity may be looked forward to in a few months. The isostatic reduction will take longer.

Physics. — *Measurements concerning the volumes of mercury menisci.* By W. H. KEESOM, Miss H. VAN DER HORST and K. W. TACONIS. (Abstract of Communication N°. 248a from the Kamerlingh Onnes Laboratory at Leiden).

(Communicated at the meeting of April 24, 1937).

The volumes of mercury menisci were measured by making RÖNTGEN shadowgraphs of menisci in tubes with radii of 9.422, 7.845, 5.700 and 3.965 mm. Plotting out volume divided by diameter ^{2} × height against the diameter of the tube, for constant height, we got straight lines. This result enabled us to derive the volumes of mercury menisci, put together in the following table:

Volumes of mercury menisci in mm^3.

h mm \ r^2 mm^2	10	20	30	40	50	60	70	80	90	100
0.7	11.9	25.0	39.0	53.4	68.4	83.8				
0.8	13.6	28.7	44.7	61.3	78.5	96.2	114.5	133.1		
0.9	15.4	32.5	50.5	69.3	88.7	108.7	129.3	150.3		
1.0	17.2	36.3	46.4	77.3	99.0	121.3	144.2	167.6		
1.1	19.1	40.2	62.4	85.4	109.4	134.0	159.2	185.0	211.3	
1.2	21.0	44.2	68.5	93.6	119.9	146.8	174.3	202.5	231.3	
1.3	23.0	48.2	74.7	102.0	130.6	159.7	189.6	220.2	251.5	
1.4	25.0	52.3	81.0	110.6	141.5	172.8	205.2	238.2	272.0	
1.5	27.1	56.5	87.4	119.4	152.6	186.2	221.2	256.8	293.0	
1.6	29.2	60.8	94.0	128.4	164.0	200.1	237.6	276.0	314.6	355.0
1.7			100.8	137.6	175.7	214.6	254.5	295.8	336.9	380.4
1.8			107.8	147.0	187.7	229.6	272.1	316.2	360.1	406.6
1.9					200.0	245.2	290.3	337.2	384.1	433.6
2.0									409.1	461.6

Physics. — *Measurements of the latent heat of tin in passing from the superconductive to the non-superconductive state at constant temperature.* By W. H. KEESOM and P. H. VAN LAER. (Abstract of Communication No. 248c from the Kamerlingh Onnes Laboratory at Leiden).

(Communicated at the meeting of April 24, 1937).

As a continuation of former preliminary experiments, we measured the latent heat of tin, when passing from the superconductive to the non-superconductive state, by increasing the external field at constant temperature. Measurements were made at the temperatures 2.971, 2.362, 2.303, 1.835, 1.239 °K. We used a massive block of tin, which again had the shape of a rotation ellipsoid, axis of rotation 17.5 cm, short axis 3.5 cm.

After the block had been cooled in a zero magnetic field, and the calorimeter box exhausted, a magnetic field was set up and gradually increased. When the first cooling of the block was observed, a constant heating current was switched on and the external field adjusted so as to keep the temperature as constant as possible.

The current was kept flowing until the moment that the heating by the current could no longer be compensated by increasing the external field. The energy applied in this way immediately gives the latent heat for the whole block at the temperature at which the measurements were made.

As the values of the transition heat thus measured agreed very well with the formula $r = -\frac{VTH}{4\pi}\frac{dH}{dT}$, which is only valid for a reversible process, we again concluded that the transition process from the superconductive to the non-superconductive state is reversible.

The disturbance of the superconductivity of the block of tin, as far as it is indicated by caloric effects, occurred at the same critical fields which are characteristic of resistance and induction phenomena.

The transition proceeded very continuously and for the larger part of the transition region linearly with the external field, which tells strongly for the assumption of a sort of intermediate state, whatever this may be.

Physics. — *Relaxation phenomena in superconductivity.* By W. H. KEESOM and P. H. VAN LAER. (Abstract of Communication No. 248d from the Kamerlingh Onnes Laboratory at Leiden).

(Communicated at the meeting of April 24, 1937).

In previous calorimetric experiments we observed that the transition from the superconductive to the non-superconductive state in the case of tin,

if effected by heating, a constant magnetic field being applied, takes place with a relaxation of several seconds (ordinarily about 30). If the transition was realised by increasing the external magnetic field, no such relaxation was observed. The measurements were carried out on a block of tin of the shape of a rotation ellipsoid provided with two cores for heating and temperature measuring. To be sure that the time-effects could not be accounted for by the complex internal structure of the first block, we repeated the experiments with a massive block of the same dimensions. The temperature records during a calorimetric experiment in all respects showed the same feature, from which we conclude that the relaxation phenomenon, mentioned above, must be characteristic of the transition process itself.

In the penetration of the magnetic field a retardation occurred of the order of tens of minutes. An analogous retardation often was observed by other experimenters in measurements of the change in induction. The penetration of the internal magnetic field and probably also the change of induction succeeds the transition so far as it is attended by a caloric effect.

We compared our results with those of other experimenters and concluded that several relaxation and hysteresis phenomena of a different kind are to be distinguished.

Astronomy. — *Mittlere Lichtkurven von langperiodischen Veränderlichen.* XXVIII. *Z Ceti.* Von A. A. NIJLAND †.

(Communicated at the meeting of April 24, 1937). *)

Instrumente *S* und *R*. Die Beobachtungen wurden alle auf *R* reduziert; die Reduktion *R*—*S* beträgt —$0^m.16$. Spektrum M 1—4e (HA **79**). Gesamtzahl der Beobachtungen 585 (von 2417082 bis 2428219). Es wurden wieder, wie in allen früheren Mitteilungen, die in zwei Instrumenten angestellten Schätzungen nur einmal gezählt. Sieben stark abweichende Schätzungen (2417225, 7770, 2422338, 2346, 3637, 5806 und 6321), in der Figur 1 eingeklämmert, wurden verworfen, und so bleiben 578 Beobachtungen für die Diskussion übrig. Der Stern ist von Mitte Februar bis Mitte Juli nicht beobachtbar, und die Lichtkurve (s. Fig. 1) zeigt Lücken, welche den Kurvenzug des öfteren unsicher machen.

Die Tabelle I gibt eine Uebersicht der benutzten Vergleichsterne.

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*) Die von NIJLAND hinterlassenen Beobachtungen der noch nicht bearbeiteten langperiodischen Veränderlichen sind im Auftrage des zeitweiligen Direktors der Sternwarte, Dr. J. VAN DER BILT, und unter dessen Direktion von dem Beamten der Sternwarte Herrn D. VAN SUYLEN, der als Gehilfe NIJLAND's auch die früheren Sterne behandelt hat, in genau derselben Weise reduziert worden. Die Resultate werden in derselben Weise wie bei den früheren Sternen in diesen Proceedings veröffentlicht werden.

TABELLE 1. Vergleichsterne.

*	BD	* — v	St.	HA 57	Grenze	H
	°			m		m
A	— 2.154	—	50.6	8.76	—	8.66
a	2.161	+0m 38s; — 1′ 10″	43.2	9.26	—	9.42
b	2.159	— 0 40; 0 0	39.1	9.79	—	9.84
c	2.164	+ 1 21; — 12 20	34.7	9.98	—	10.30
γ	—	+ 1 25; + 3 20	33.6	—	—	10.42
d	2.166	+ 1 37; — 9 20	26.7	—	—	11.13
e	—	+ 0 18; — 3 30	20.4	11.80	—	11.79
f	—	— 0 44; — 3 40	12.8	12 82	m 13.40	12.58
g	—	— 0 48; — 7 20	10.3	[13.80]	—	12.84
h	—	— 0 42; — 2 0	6.1	—	—	13.28
j	—	+ 0 5; + 3 20	4.4	—	13.57	13.45
k	—	— 0 1; + 3 50	0.0	—	14.00	13.91

Stern *f* wurde zwölfmal, Stern *j* sechsmal, Stern *k* fünfmal an die Grenze von *R* angeschlossen; für *f* folgt daraus die sehr unsichere Helligkeit $13^m.40$. Der Wert $13^m.80$ für *g* aus *Harv. Ann* 57 musste unberücksichtigt bleiben. Der Stufenwert ist $0^m.104$.

Es liegen 37 Schätzungen der Farbe vor (Tabelle IIa und II*b*). Das allgemeine Mittel ist $2^c.06$.

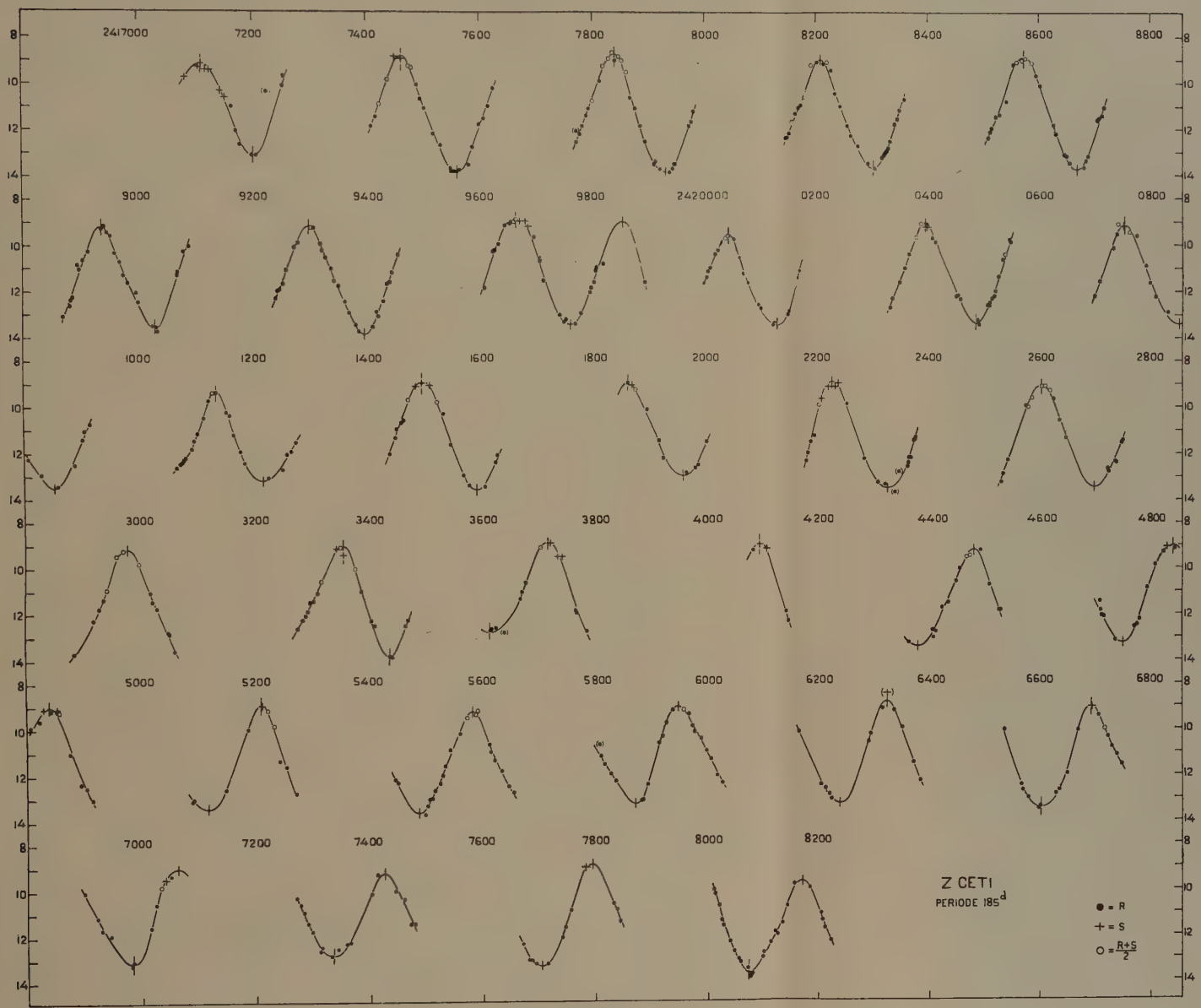
TABELLEN IIa und II*b*. Farbenschätzungen.

Zeitraum	n	Farbe	Grösse	n	Farbe
		c	m		c
2417117 — 2417935	11	1.41	8.84	11	1.86
8183 — 9668	13	2.27	9.10	13	2.35
2420749 — 7780	13	2 42	10.06	13	1.96
	37			37	

Die Figur 1 enthält die Beobachtungen, alle auf *R* reduziert. Die Reihe der Abweichungen (Beobachtung minus Kurve) zeigt 169 Plus-, 192 Minuszeichen, 217 Nullwerte, 152 Zeichenfolgen, 208 Zeichenwechsel. Das Mittel der absoluten Werte der Abweichungen ist $0^m.087$.

Ein Einfluss des Mondscheines auf die Helligkeitsschätzung ist nicht nachweisbar. Es verteilen sich auf 159 bei Mondschein angestellte Beobachtungen die Abweichungen wie folgt: 46 Plus-, 52 Minuszeichen, 61 Nullwerte.

Die Tabelle III enthält die aus der Kurve abgelesenen Epochen der


2417000
7200
7400
7600
7800
8000
8200
8400
8600
8800
9000
9200
9400
9600
9800
2420000
0200
0400
0600
0800
1000
1200
1400
1600
1800
2000
2200
2400
2600
2800
3000
3200
3400
3600
3800
4000
4200
4400
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4800
5000
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5400
5600
5800
6000
6200
6400
6600
6800
7000
7200
7400
7600
7800
8000
8200
Z CETI
PERIODE 185d
● = R
+ = S
○ = R+S/2

Fig. 1.

TABELLE III.

E	Minima *m*					E	Maxima *M*				
	B	*v*	*R*	*B*—*R*	*B*—*F*		*B*	*v*	*R*	*B*—*R*	*B*—*F*
	241	m					241	m			
— 30	7201	13.1	7176	+ 25	+ 13	— 30	7112	9.1	7083	+ 29	+ 17
— 28	7566	13.8	7545	+ 21	+ 9	— 28	7462	8.7	7452	+ 10	— 2
— 26	7928	13.8	7914	+ 14	+ 4	— 26	7840	8.7	7821	+ 19	+ 9
— 24	8300	13.7	8284	+ 16	+ 8	— 24	8209	8.9	8191	+ 18	+ 10
— 22	8667	13.7	8653	+ 14	+ 9	— 22	8572	8.9	8560	+ 12	+ 7
— 20	9028	13.6	9022	+ 6	+ 4	— 20	8936	9.2	8929	+ 7	+ 5
— 18	9394	13.8	9391	+ 3	+ 5	— 18	9298	9.2	9298	0	+ 2
— 16	9758	13.4	9760	— 2	+ 4	— 16	9663	8.8	9667	— 4	+ 2
— 14	242 0126	13.4	0130	— 4	+ 6	— 14	242 0039	9.6	0037	+ 2	+ 8
— 12	0484	13.4	0499	— 15	— 2	— 12	0392	9.0	0406	— 14	— 1
— 10	0850	13.4	0868	— 18	— 2	— 10	0751	9.2	0775	— 24	— 8
— 8	1216	13.2	1237	— 21	— 3	— 8	1132	9.3	1144	— 12	+ 6
— 6	1590	13.6	1606	— 16	+ 4	— 6	1494	8.9	1513	— 19	+ 1
— 4	1956	12.9	1976	— 20	0	— 4	1860	8.9	1883	— 23	— 3
— 2	2324	13.5	2345	— 21	— 1	— 2	2226	8.9	2252	— 26	— 6
0	2696	13.5	2714	— 18	0	0	2602	9.1	2621	— 19	— 1
+ 4	3436	13.8	3452	— 16	— 3	+ 2	2976	9.2	2990	— 14	+ 2
+ 5	3611	12.8	3637	— 26	— 15	+ 4	3354	9.0	3359	— 5	+ 8
+ 9	4378	13.4	4375	+ 3	+ 7	+ 6	3715	8.9	3729	— 14	— 5
+ 11	4748	13.3	4745	+ 3	+ 3	+ 10	4481	9.2	4467	+ 14	+ 15
+ 13	5116	13.4	5114	+ 2	— 2	+ 12	4839	9.0	4836	+ 3	+ 1
+ 15	5485	13.6	5483	+ 2	— 5	+ 14	5209	8.9	5205	+ 4	— 2
+ 17	5870	13.2	5852	+ 18	+ 8	+ 16	5581	9.2	5575	+ 6	— 2
+ 19	6237	13.2	6221	+ 16	+ 5	+ 18	5946	9.0	5944	+ 2	— 9
+ 21	6600	13.4	6591	+ 9	— 3	+ 20	6322	8.8	6313	+ 9	— 3
+ 23	6982	13.1	6960	+ 22	+ 10	+ 22	6691	9.1	6682	+ 9	— 3
+ 25	7334	12.8	7329	+ 5	— 6	+ 24	7063	9.0	7051	+ 12	+ 1
+ 27	7703	13.3	7698	+ 5	— 4	+ 26	7425	9.3	7421	+ 4	— 6
+ 29	8071	13.7	8067	+ 4	— 2	+ 28	7794	8.9	7790	+ 4	— 3
		13.41				+ 30	8172	9.6	8159	+ 13	+ 9
								9.05			

Minima m und der Maxima M. Die Spalte R wurde mit den einfachen Elementen:

$$2422621^d + 184^d.6\,E \text{ (für die Maxima)}$$

und $$2422714 + 184\ .6\,E \text{ (für die Minima)}$$

gerechnet.

Die übrigbleibenden $B—R$ zeigen einen systematischen Charakter; es wurde für Minima und Maxima zusammen auf graphischem Wege ein Sinusglied abgeleitet, und die definitiven Elemente F lauten dann:

$$\left.\begin{array}{l}\text{Maximum: } 2422617\\ \text{Minimum: } 2422710\end{array}\right\} + 184^d.6\,E - 16^d \sin 7^0\,(E+17).$$

Für $\frac{M-m}{p}$ findet man 0.496.

SCHNELLER's Katalog für 1937 gibt den Periodenwert $184^d.6$.

Die extremen Werte des Lichtwechsels sind:

$$\left.\begin{array}{l}\text{Maximum: } 9^m.05 \pm 0^m.04\\ \text{Minimum: } 13\ .41 \pm 0\ .05\end{array}\right\} \text{(m.F.)}.$$

Die Amplitude beträgt somit $4^m.36$.

Es wurde in der üblichen Weise der mittlere Verlauf der Lichtkurve in der Umgebung der Hauptphasen bestimmt. Die sehr unsicheren Maxima 2419852 und 2424093 blieben hierbei unberücksichtigt. Die beiden Teil-

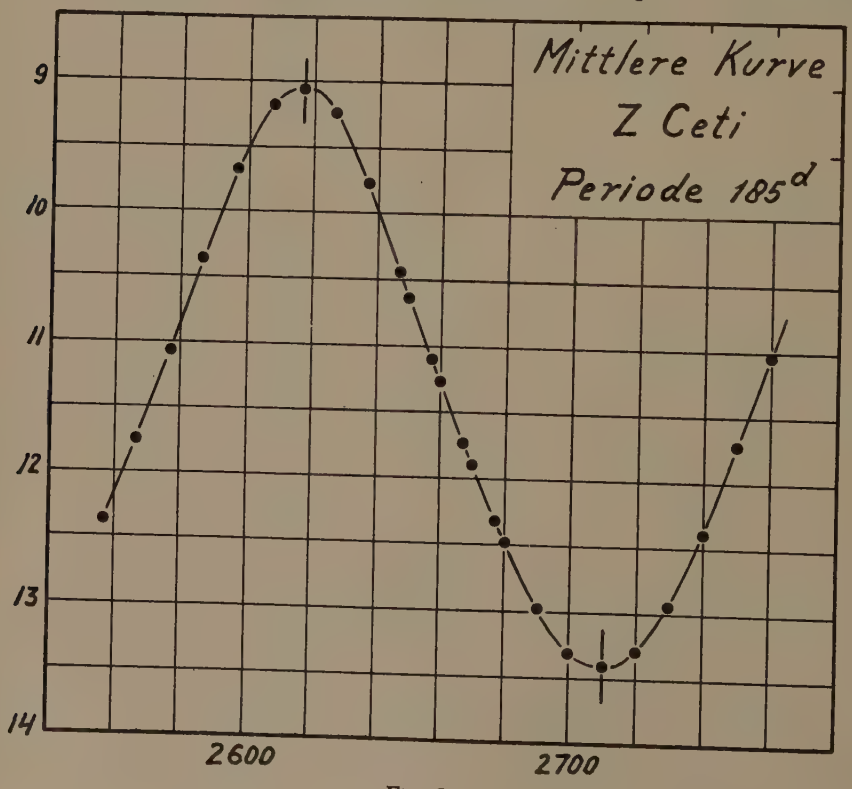

Fig. 2.

kurven schliessen sich, wie aus der Figur 2 ersichtlich, vorzüglich an einander an, und liefern zusammen eine glatt verlaufende mittlere Kurve (Tabelle IV).

In der Tabelle IV wurde 2617 als Nullpunkt der Phase genommen.

TABELLE IV. Die mittlere Kurve.

Phase	v	Phase	v	Phase	v	Phase	v
-60^d	12.36^m	0^d	9.05^m	$+60^d$	12.32^m	$+120^d$	12.56^m
−50	11.74	+10	9.23	+70	12.83	+130	11.95
−40	11.06	+20	9.77	+80	13.22	+140	11.26
−30	10.35	+30	10.45	+90	13.39		
−20	9.67	+40	11.10	+100	13.35		
−10	9.20	+50	11.73	+110	13.06		

Die Streuung in der Nähe von 50^d erreicht die Werte:

	m	M	Mittel
im aufsteigenden Aste:	$0^m.432$	$0^m.243$	$0^m.337$
im absteigenden Aste:	0 .309	0 .270	0 .289
Mittel	0 .370	0 .256	

Die Streuung ist kleiner beim Maximum und grösser im aufsteigenden Aste.

Das Verhältnis der Streuungen $0^m.337$ und $0^m.289$ ist 1.17, das Verhältnis der durchschnittlichen Geschwindigkeiten des Lichtwechsels bei Auf- und Abstieg ist 1.02.

Utrecht, Dezember 1936.

Astronomy. — *Mittlere Lichtkurven von langperiodischen Veränderlichen.* XXIX. *U Persei.* Von A. A. NIJLAND †.

(Communicated at the meeting of April 24, 1937).

Instrumente *S* und *R*. Die Beobachtungen wurden alle auf *R* reduziert; die Reduktion *R*—*S* beträgt $-0^m.16$. Spektrum M 6—7e (HA 79). Gesamtzahl der Beobachtungen 818 (von 2416836 bis 2428278). Wie in früheren Publikationen wurden die in zwei Instrumenten angestellten Schätzungen nur einmal gezählt. Vier stark abweichende Schätzungen (2416919, 2423997, 5620 und 7374), in der Figur 1 eingeklämmert, wurden

verworfen, und so bleiben 814 Beobachtungen für die Diskussion übrig.

Karte: HAGEN, *Atlas Stell. var. Series III.*

Die Tabelle I gibt eine Uebersicht der benutzten Vergleichsterne.

TABELLE I. Vergleichsterne.

*	HA 57	HAGEN	BD	St.	PD *red*	HA 74	HA 91	MITCHELL	*H*
A	—	—	+53°.419	42.2	—	—		—	7m.24
a	*a*	3	53.440	36.7	7m.94	7m.80	7m.50G5	—	7.80
β	—	—	54.452	32.7	—	8.03	7.66 F8	—	8.21
b	*c*	7	54.438	26.8	—	8.90	7.96A0	8m.8	8.82
c	—	11	54.436	18.5	—	—		—	9.68
γ	*g*	24	54.435	17.0	—	9.91		10.1	9.84
d	—	19		14.6	—	—		—	10.08
e	—	33		11.9	—	—		—	10.37
λ	*l*	34		10.2	—	10.56		10.6	10.54
f	*n*	37		8.9	—	10.77		10 8	10.66
g	*o*	42		5.1	Grenze	11.02		11.1	11.05
h	*p*	50		0.0	11.44	11.54		11.6	11.59

Stern *h* wurde viermal (2417206, 7230, 2426001, 6024) an die Grenze von *S* angeschlossen. Der Stufenwert ist 0m.103.

Es liegen 116 Schätzungen der Farbe vor (Tabelle II*a* und II*b*). Das allgemeine Mittel ist 3c.16.

TABELLEN II*a* und II*b*. Farbenschätzungen.

Zeitraum	*n*	Farbe	Grösse	*n*	Farbe
241 6931 — 7392	20	3c.00	7m.43	13	2c.31
7399 — 7638	20	3.20	7.77	13	2.54
7646 — 8010	20	2.60	7.94	13	2.92
8024 — 8440	20	3.15	8.18	13	3.08
8555 — 9616	20	3.45	8.45	13	2.85
9658 — 242 7955	16	2.75	8.77	13	3.08
	116		9.16	13	3.69
			9.45	13	3.77
			10.15	12	4.17
				116	

A. A. NIJLAND †: MITTLERE LICHTKURVEN VON LANGPERIODISCHEN VERÄNDERLICHEN. XXIX. U PERSEI.

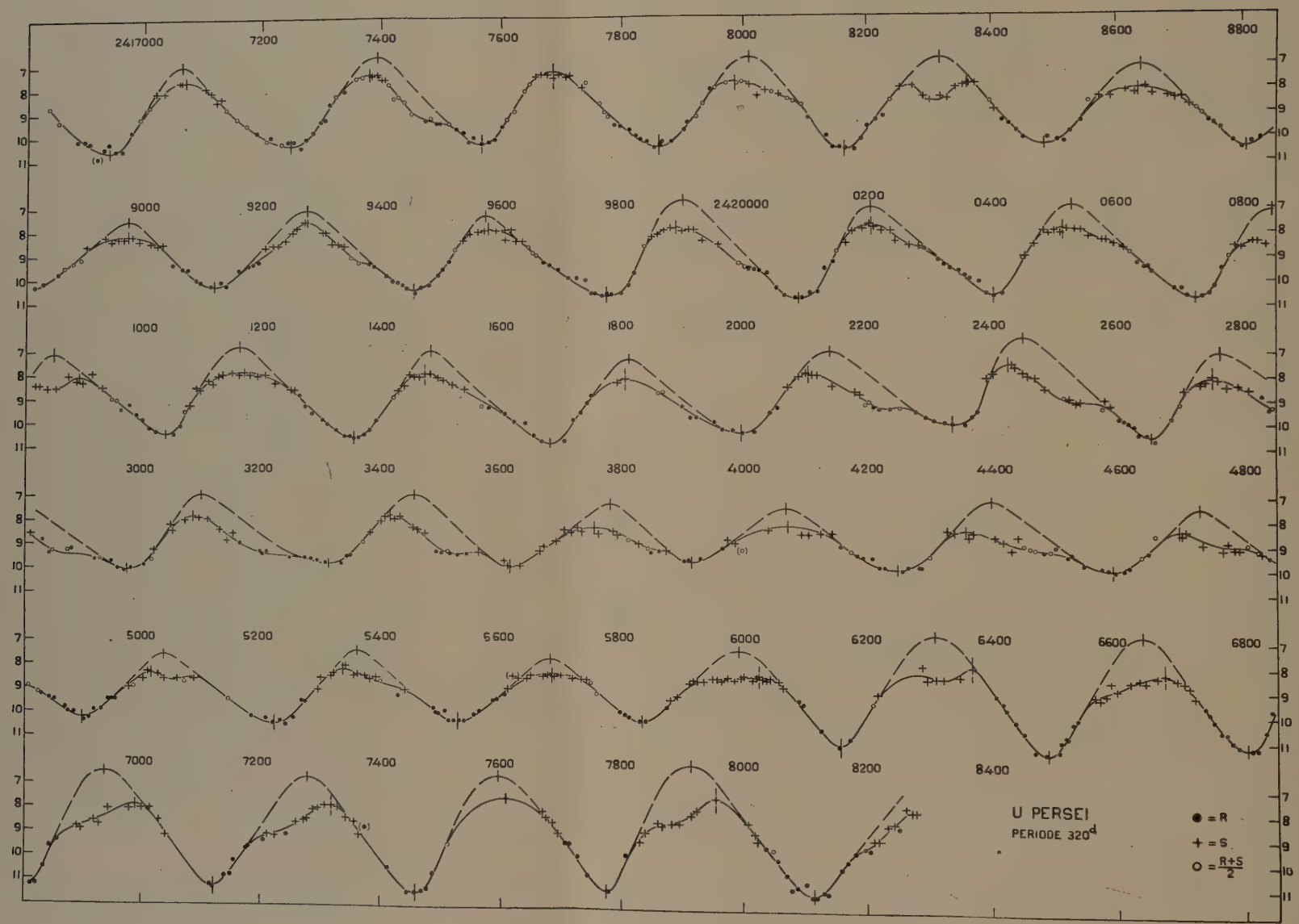

Fig. 1.

Die Figur 1 enthält die Beobachtungen, alle auf R reduziert. Die Reihe der Abweichungen (Beobachtung minus Kurve) zeigt 271 Plus-, 299 Minuszeichen, 244 Nullwerte, 260 Zeichenfolgen, 309 Zeichenwechsel. Das Mittel der absoluten Werte der Abweichungen ist $0^m.083$.

Ein Einfluss des Mondscheines auf die Helligkeitsschätzung ist nicht nachweisbar. Es verteilen sich auf 256 bei Mondschein angestellte Beobachtungen die Abweichungen wie folgt: 92 Plus-, 93 Minuszeichen, 71 Nullwerte.

Die Tabelle III enthält die aus der Kurve abgelesenen Epochen der Minima m und der Maxima M. Die Spalte R wurde mit den einfachen Elementen:

$$2422334^d + 320^d.0\,E \text{ (für die Minima)}$$

und

$$2422473 + 320\ .0\,E \text{ (für die Maxima)}$$

gerechnet.

SCHNELLER's Katalog für 1937 gibt den Periodenwert 328^d.

Die extremen Werte des Lichtwechsels sind:

$$\left.\begin{array}{l}\text{Minimum: } 10^m.56 \pm 0^m.080\\ \text{Maximum: } 7\ .96 \pm 0\ .052\end{array}\right\}\text{(m.F.)}.$$

Die Amplitude beträgt somit $2^m.60$.

Auch für U Persei wurde wieder der mittlere Verlauf der Lichtkurve in der Nähe der beiden Hauptphasen durch Ablesung der Helligkeit für je 10^d abgeleitet. Das sehr unsichere Maximum 2427608 blieb hierbei unberücksichtigt.

Die beiden Teilkurven schliessen sich einander, wie aus der Figur 2 ersichtlich, nicht befriedigend an. Sie liefern zusammen die mittlere

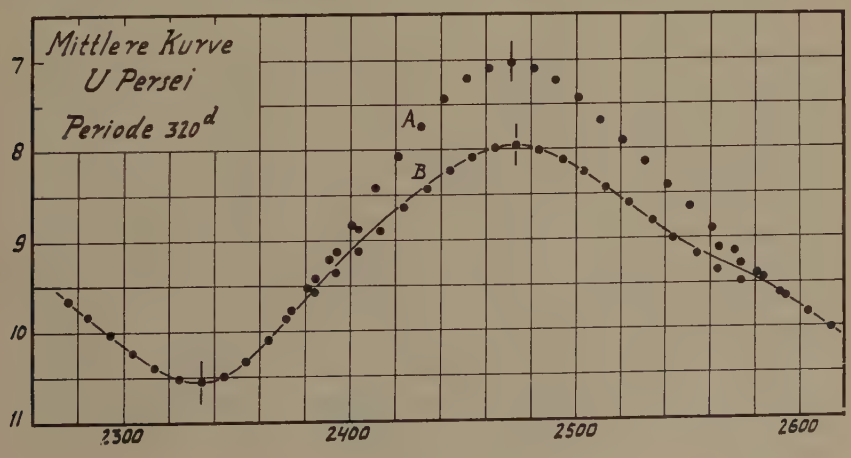

Fig. 2.

Kurve B (Tabelle IV); s. auch $V\,20 = ST\ Cygni$ (Proc. 38, 970). Wird auch bei U Persei in der üblichen Weise die Kurve von der Störung

TABELLE III.

E	Minima *m*				Maxima *M*			
	B	*v*	*R*	*B*—*R*	*B*	*v*	*R*	*B*—*R*
	241	m			241	m		
— 17	6940	10.6	6894	+ 46	7070	7.6	7033	+ 37
16	7248	10.4	7214	+ 34	7380	7.4	7353	+ 27
15	7568	10.3	7534	+ 34	7685	7.3	7673	+ 12
14	7862	10.4	7854	+ 8	7986	7.7	7993	— 7
13	8165	10.5	8174	— 9	8362	7.8	8313	+ 49
12	8485	10.4	8494	— 9	8635	8.1	8633	+ 2
11	8806	10.5	8814	— 8	8974	8.2	8953	+ 21
10	9121	10.3	9134	— 13	9276	7.6	9273	+ 3
9	9455	10.4	9454	+ 1	9579	7.9	9593	— 14
8	9775	10.7	9774	+ 1	9890	7.8	9913	— 23
	242				242			
7	0090	10.8	0094	— 4	0208	7.8	0233	— 25
6	0403	10.7	0414	— 11	0513	7.9	0553	— 40
5	0725	10.8	0734	— 9	0892	8.1	0873	+ 19
4	1040	10.4	1054	— 14	1173	7.9	1193	— 20
3	1356	10.6	1374	— 18	1475	8.0	1513	— 38
2	1680	10.8	1694	— 14	1806	8.2	1833	— 27
— 1	1996	10.4	2014	— 18	2105	7.9	2153	— 48
0	2337	10.0	2334	+ 3	2425	7.5	2473	— 48
+ 1	2654	10.6	2654	0	2752	8.1	2793	— 41
2	2974	10.1	2974	0	3086	7.9	3113	— 27
3	3314	9.8	3294	+ 20	3417	7.8	3433	— 16
4	3615	9.9	3614	+ 1	3755	8.3	3753	+ 2
5	3915	9.7	3934	— 19	4071	8.2	4073	— 2
6	4249	10.0	4254	— 5	4359	8.4	4393	— 34
7	4592	10.0	4574	+ 18	4698	8.4	4713	— 15
8	4898	10.3	4894	+ 4	5016	8.4	5033	— 17
9	5224	10.5	5214	+ 10	5338	8.2	5353	— 15
10	5530	10.3	5534	— 4	5684	8.3	5673	+ 11
11	5835	10.3	5854	— 19	6025	8.4	5993	+ 32
12	6158	11.2	6174	— 16	6368	8.0	6313	+ 55
13	6487	11.6	6494	— 7	6673	8.3	6633	+ 40
14	6804	11.2	6814	— 10	6990	7.9	6953	+ 37
15	7121	11.3	7134	— 13	7319	7.9	7273	+ 46
16	7460	11.5	7454	+ 6	[7608]	[7.5]	—	—
17	7775	11.3	7774	+ 1	7954	7.5	7913	+ 41
+ 18	8114	11.4	8094	+ 20		7.96		± 26
		10.56		± 12				

TABELLE IV. Die mittlere Kurve.

Phase	v	Phase	v	Phase	v	Phase	v	Phase	v
d	m	d	m	d	m	d	m	d	m
− 60	9.65	+ 10	10.50	+ 80	8.80	+ 150	8.03	+ 220	9.14
− 50	9.84	+ 20	10.35	+ 90	8.58	+ 160	8.14	+ 230	9.25
− 40	10.03	+ 30	10.10	+ 100	8.39	+ 170	8.27	+ 240	9.36
− 30	10.23	+ 40	9.81	+ 110	8.22	+ 180	8.45	+ 250	9.50
− 20	10.40	+ 50	9.52	+ 120	8.08	+ 190	8.63	+ 260	9.65
− 10	10.51	+ 60	9.26	+ 130	7.97	+ 200	8.81	+ 270	9.84
0	10.56	+ 70	9.01	+ 140	7.96	+ 210	8.99	+ 280	10.03

TABELLE V. Ungestörte Maxima.

E	B	v	R	$B-R$	E	B	v	R	$B-R$
	241	m					m		
− 17	7064	7.0	7031	+ 33	0	2448	6.4	2471	− 23
16	7392	6.7	7351	+ 41	+ 1	2764	7.0	2791	− 27
15	—	—	—	—	2	3100	7.0	3111	− 11
14	8010	6.7	7991	+ 19	3	3458	7.0	3431	+ 27
13	8318	6.8	8311	+ 7	4	3782	7.3	3751	+ 31
12	8639	7.1	8631	+ 8	5	4068	7.5	4071	− 3
11	8974	7.5	8951	+ 23	6	4398	7.2	4391	+ 7
10	9276	7.1	9271	+ 5	7	4730	7.5	4711	+ 19
9	9574	7.3	9591	− 17	8	5036	7.6	5031	+ 5
8	9901	6.7	9911	− 10	9	5362	7.4	5351	+ 11
	242								
7	0207	7.0	0231	− 24	10	5681	7.8	5671	+ 10
6	0527	7.0	0551	− 24	11	5991	7.4	5991	0
5	0847	7.1	0871	− 24	12	6308	6.7	6311	− 3
4	1164	6.8	1191	− 27	13	6638	6.7	6631	+ 7
3	1485	7.0	1511	− 26	14	6936	6.5	6951	− 15
2	1814	7.3	1831	− 17	15	7279	6.7	7271	+ 8
− 1	2140	7.0	2151	− 11	16	[7596]	[6.7]	—	—
					+ 17	7914	6.2	7911	+ 3
							7.03		

befreit, so entstehen die ungestörten Maxima, welche (Tabelle V) mit den einfachen Elementen R:

$$2422471^d + 320^d.0\,E$$

verglichen wurden.

Die maximale „ungestörte" Helligkeit ist:

$$v = 7^m.03 \pm 0^m.063 \text{ (m.F.)}.$$

Die Teilkurve A der ungestörten Maxima (Fig. 2) schliesst sich derjenigen der Minima vollständig an. Für die Schiefe der ungestörten Kurven findet man: $\frac{M-m}{p} = 0.428$.

Utrecht, Februar 1937.

Astronomy. — *Mittlere Lichtkurven von langperiodischen Veränderlichen. XXX. S Lyncis.* Von A. A. NIJLAND †.

(Communicated at the meeting of April 24, 1937).

Instrumente S und R. Die Beobachtungen wurden alle auf R reduziert; die Reduktion R—S beträgt —$0^m.18$. Spektrum M6e (HA**79**). Gesamtzahl der Beobachtungen 567 (von 2417253 bis 2428327). Es wurden wieder, wie in alle früheren Mitteilungen, die in zwei Instrumenten angestellten Schätzungen nur einmal gezählt.

Fünf stark abweichende Schätzungen, in der Figur I eingeklämmert, wurden verworfen, und so bleiben 562 Beobachtungen für die Diskussion übrig.

Karte: HAGEN, *Atlas Stell. var. Series VI.*

Die Tabelle I gibt eine Uebersicht der benutzten Vergleichsterne. Die Sterne h und g wurden nur ausnahmsweise verwendet. Die Sterne k, l, m und n wurden 8-, bezw. 21-, 16- und 14-mal an die Grenze von R angeschlossen: die sich hieraus ergebenden Helligkeiten sind: $k = 13^m.72$, $l = 13^m.58$, $m = 13^m.68$, $n = 14^m.08$. Der Wert für k ist m. E. sicher zu schwach. Der Stufenwert ist $0^m.094$.

Es liegen 40 Schätzungen der Farbe vor (Tabelle IIa und II*b*). Das allgemeine Mittel ist $2^c.46$.

Die Figur 1 enthält die Beobachtungen, alle auf R reduziert. Die Reihe der Abweichungen (Beobachtung minus Kurve) zeigt 171 Plus-, 191 Minuszeichen, 200 Nullwerte, 138 Zeichenfolgen, 223 Zeichenwechsel. Das Mittel der absolute Werte der Abweichungen ist $0^m.087$.

Ein Einfluss des Mondscheines auf die Helligkeitsschätzung ist nicht nachweisbar. Es verteilen sich auf 128 bei Mondschein angestellte Beob-

A. A. NIJLAND †: MITTLERE LICHTKURVEN VON LANGPERIODISCHEN VERÄNDERLICHEN. XXX. S LYNCIS.

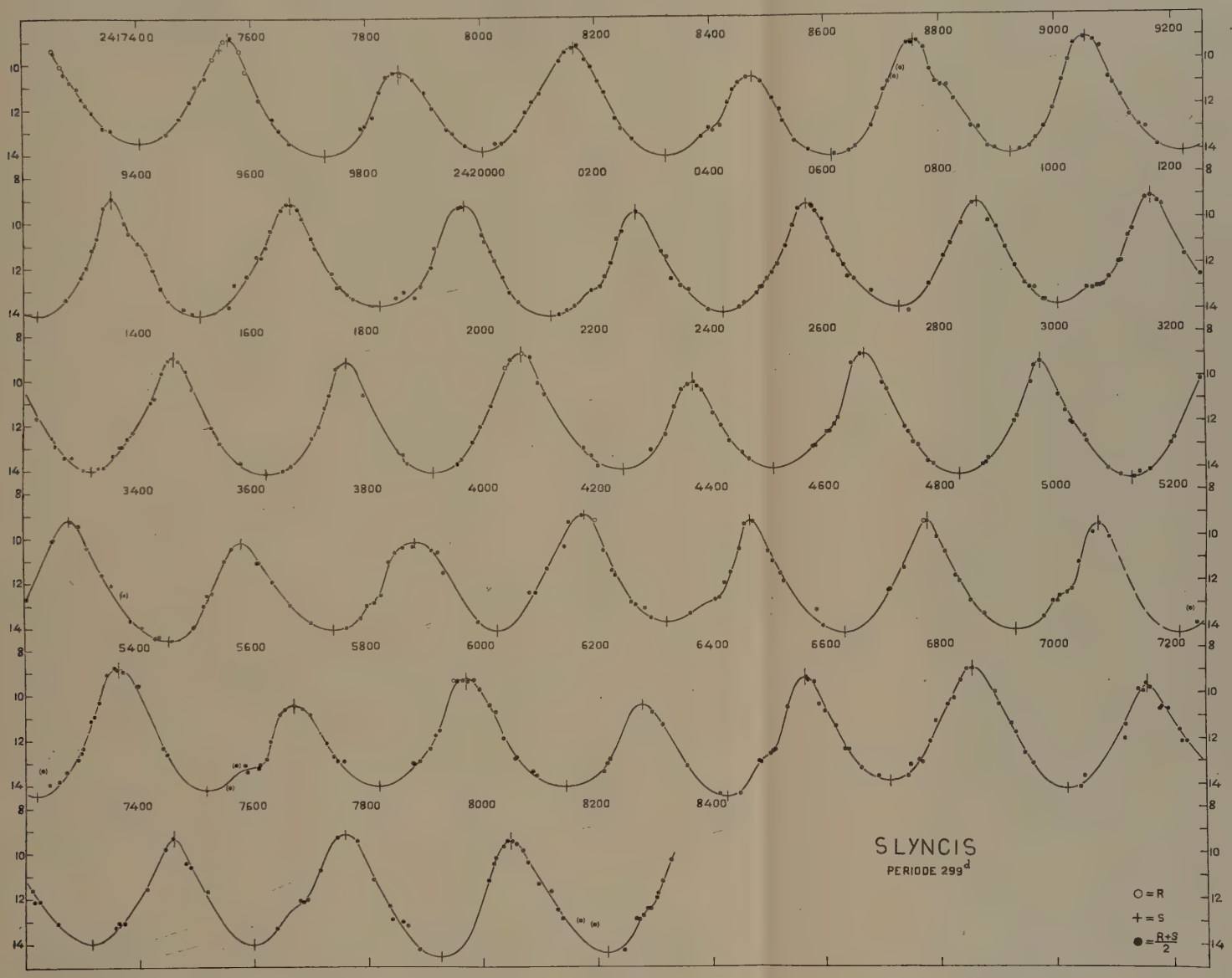

Fig. 1.

TABELLE I. Vergleichsterne.

*	HAGEN	BD * — v	St.	HA 57	Grenze	H
B	3	$+58^{\circ}.954$	62.2	$8^{m}.45$	—	$8^{m}.56$
A	10	+58.964	55.4	9.26	—	9.20
a	13	+58.960	52.0	9.71	—	9.52
b	12	+58.965	46.7	9.87	—	10.01
c	16		43.3	—	—	10.33
d	22		37.2	—	—	10.90
e	25		37.1	—	—	10.91
f	28		32.2	—	—	11.37
g	—	$+11^{s}-7'10''$?	27.8	—	—	11.78
h	36		23.1	—	—	12.22
j	37		18 8	—	—	12.62
k	39		13.0	—	$[13^{m}.72]$	$13^{m}.17$
l	—	$+2^{s}-20''$	9.4	—	13.58	13.60
m	—	+12 +20	5.1	—	13.68	13.90
n	—	+ 6 — 59	0.0	—	14.08	14.38

TABELLE IIa und II*b*. Farbenschätzungen.

Zeitraum	n	Farbe	Grösse	n	Farbe
		$2^{c}.50$	$8^{m}.96$		$2^{c}.46$
2417253 — 2418751	13			13	
2418761 — 2424466	13	2.50	9.29	13	2.31
2425343 — 2428057	14	2.39	9.62	14	2 61
	40			40	

achtungen die Abweichungen wie folgt: 41 Plus-, 38 Minuszeichen, 49 Nullwerte.

Die Tabelle III enthält die aus der Kurve abgelesenen Epochen der Minima m und der Maxima M. Die Spalte R wurde mit den einfachen Elementen:

$$2422522^{d} + 299^{d}.2\,E \text{ für die Minima)}$$

und

$$2422667 + 299.2\,E \text{ (für die Maxima)}$$

gerechnet.

Die übrigbleibenden B—R sind zu gross. Da es nicht gelungen ist,

TABELLE III.

E	Minima m				Maxima M			
	B	v	R	$B-R$	B	v	R	$B-R$
	241	m			241	m		
− 17	7406	13.5	7436	− 30	7560	8.9	7581	− 21
− 16	7728	14.1	7735	− 7	7857	10.3	7880	− 23
− 15	8005	13.9	8034	− 29	8163	9.3	8179	− 16
− 14	8324	14.1	8333	− 9	8474	10.6	8478	− 4
− 13	8615	14.2	8632	− 17	8756	9.1	8777	− 21
− 12	8924	14.1	8932	− 8	9054	9.0	9077	− 23
− 11	9221	14.1	9231	− 10	9355	9.0	9376	− 21
− 10	9510	14.2	9530	− 20	9668	9.2	9675	− 7
− 9	9824	13.8	9829	− 5	9970	9.3	9974	− 4
	242				242			
− 8	0122	14.3	0128	− 6	0270	9.6	0273	− 3
− 7	0422	14.1	0428	− 6	0570	9.3	0573	− 3
− 6	0730	13.9	0727	+ 3	0866	9.3	0872	− 6
− 5	1005	13.8	1026	− 21	1164	9.1	1171	− 7
− 4	1320	14.0	1325	− 5	1461	9.0	1470	− 9
− 3	1625	14.2	1624	+ 1	1764	9.3	1769	− 5
− 2	1916	14.2	1924	− 8	2070	8.9	2069	+ 1
− 1	2247	14.0	2223	− 24	2367	10.2	2368	− 1
0	2511	14.0	2522	− 11	2668	9.0	2667	+ 1
+ 1	2833	14.3	2821	+ 12	2972	9.4	2966	+ 6
+ 2	3130	14.5	3120	+ 10	3280	9.2	3265	+ 15
+ 3	3453	14.6	3420	+ 33	3580	10.3	3565	+ 15
+ 4	3740	14.1	3719	+ 21	3883	10.2	3864	+ 19
+ 5	4026	14.2	4018	+ 8	4178	9.0	4163	+ 15
+ 6	4322	13.8	4317	+ 5	4467	9.3	4462	+ 5
+ 7	4634	14.3	4616	+ 18	4776	9.4	4761	+ 15
+ 8	4929	14.2	4916	+ 13	5071	9.5	5061	+ 10
+ 9	5211	14.3	5215	− 4	5364	8.8	5360	+ 4
+ 10	5518	14.2	5514	+ 4	5672	10.5	5659	+ 13
+ 11	5820	14.0	5813	+ 7	5971	9.3	5958	+ 13
+ 12	6145	14.0	6112	+ 33	6279	10.5	6257	+ 22
+ 13	6426	14.5	6412	+ 14	6564	9.3	6557	+ 7
+ 14	6712	13.8	6711	+ 1	6854	8.9	6856	− 2
+ 15	7018	14.2	7010	+ 8	7156	9.7	7155	+ 1
+ 16	7316	14.0	7309	+ 7	7460	9.4	7454	+ 6
+ 17	7600	14.0	7608	− 8	7760	9.2	7753	+ 7
+ 18	7926	14.6	7908	+ 18	8048	9.5	8053	− 5
+ 19	8217	14.4	8207	+ 10		9.44		± 10
		14.12		± 12				

hier mit einfachen Mitteln viel zu verbessern, habe ich mich mit den oben angegebenen Elementen zufrieden gestellt.

Für $\frac{M-m}{p}$ findet man 0.485.

SCHNELLER's Katalog für 1937 gibt den Periodenwert $297^{d}.3$.

Die extremen Werte des Lichtwechsels sind:

$$\left.\begin{array}{l} \text{Minimum: } 14^{m}.12 \pm 0^{m}.04 \\ \text{Maximum: } 9\ .44 \pm 0\ .08 \end{array}\right\} (\text{m.F.}).$$

Die Amplitude beträgt somit $4^{m}.68$.

Es wurde wieder der mittlere Verlauf der Lichtkurve in der Umgebung der beiden Hauptphasen durch Ablesung der Helligkeit für je 10^{d} abgeleitet. Die beiden Teilkurven schliessen sich im Aufstieg etwas weniger genau als sonst an einander an (s. die Figur 2), und geben zusammen den Verlauf der mittleren Kurve (Tabelle IV).

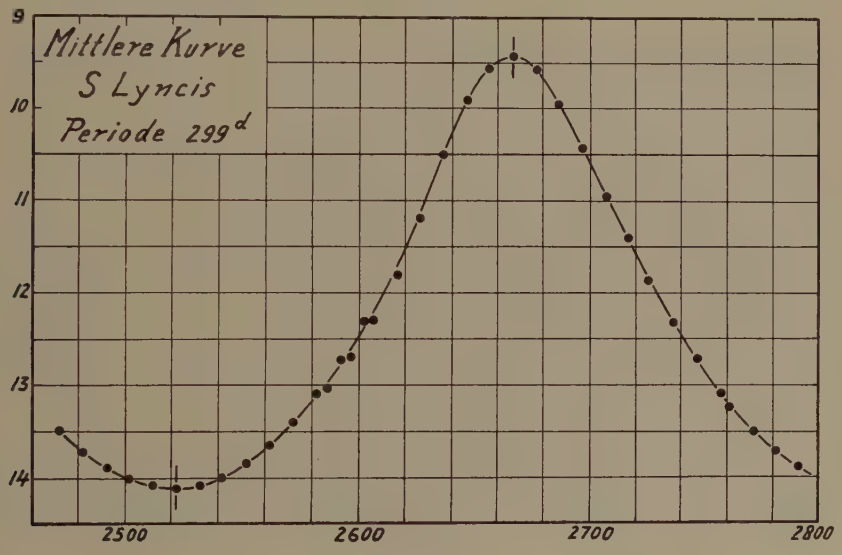

Fig. 2.

TABELLE IV. Die mittlere Lichtkurve.

Phase	v	Phase	v	Phase	v	Phase	v	Phase	v
d	m	d	m	d	m	d	m	d	m
— 50	13.49	+ 20	14.00	+ 90	11 95	+ 160	9.74	+ 230	12.91
— 40	13.72	+ 30	13.85	+ 100	11.41	+ 170	10.21	+ 240	13.26
— 30	13.90	+ 40	13.65	+ 110	10.70	+ 180	10.70	+ 250	13.52
— 20	14.01	+ 50	13.40	+ 120	10.16	+ 190	11.20	+ 260	13.74
— 10	14 07	+ 60	13 10	+ 130	9.67	+ 200	11 .66		
0	14.12	+ 70	12.77	+ 140	9.47	+ 210	12.12		
+ 10	14.08	+ 80	12.37	+ 150	9.47	+ 220	12.53		

Die Streuung in der Nähe von 80^d erreicht die Werte:

	m	M	Mittel
im aufsteigenden Aste:	$0^m.308$	$0^m.354$	$0^m.331$
im absteigenden Aste:	0 .211	0 .305	0 .258
Mittel	0 .259	0 .329	

Die Streuung ist wieder grösser beim Maximum, und grösser im aufsteigenden Aste. Das Verhältnis der Streuungen 0.331 und 0.258 ist 1.28, das Verhältnis der durchschnittlichen Geschwindigkeiten des Lichtwechsels bei Auf- und Abstieg ist 1.06.

Utrecht, April 1937.

Chemistry. — *Flow potentials on platinum.* (Preliminary communication). By H. R. KRUYT and J. OOSTERMAN.

(Communicated at the meeting of April 24, 1937).

Flow potentials occur if a liquid, flowing along a wall, carries ions from the double layer which is situated on the boundary wall-liquid. In principle, therefore, we may also expect flow potentials on a platinum capillary. They are, moreover, described by QUINCKE [1]). However, the influence of the conducting wall should be taken into consideration [2]).

For the technique of flow potential measurements we refer to the publication of KRUYT and RUYSSEN [3]).

Already the very first measurements, performed on the system platinum-twice distilled water made us distinguish the following deviations from the more normal systems isolator (glass, quartz, etc.)-aqueous solution:

a. The potential which sets in is not strictly constant, as e.g. is the case with Jena glass 397^{III} within 0,1 %.

b. After elimination of the difference in pressure on the ends of the platinum capillary, the E.M.F. decreases slowly (in case of glass it decreases, within the time required for an E. M. F. measurement, to the so-called electrode-potential).

c. The electrode-potential (the potential difference which we measure if no liquid flow occurs and which in normal cases has its origin in the inequality of the two potential differences platinum electrode-liquid) shows large variations, in contrast with the corresponding potential difference in the system glass-water.

---

[1]) G. QUINCKE, Ann. Physik (POGGENDORFF) **107**, 1 (1859); **110**, 38 (1860).

[2]) M. v. SMOLUCHOWSKI in GRAETZ's Handbuch der Elektrizität und des Magnetismus, II, p. 394 (1921).

[3]) H. R. KRUYT and R. RUYSSEN, Proc. Royal Acad. Amsterdam, **37**, 498 (1934).

These deviations are due to wall conductivity. For the occurrence of this wall conductivity it is required that the ions, being liberated at the ends of the capillary, are discharged on the metal wall. As a result of this discharge, the ends of the capillary are polarized into hydrogen- and oxygen-electrodes.

The E. M. F., which is measured as apparent flow potential, may be considered to be composed as follows:

Potential difference platinum electrode-liquid + pot. difference liquid-hydrogen electrode (in one vessel) + pot. difference oxygen electrode-liquid (in the other vessel) + pot. difference liquid-platinum electrode. The hydrogen- and oxygen electrodes have a conductive connection by means of the capillary wall.

From these observations we may draw the following conclusions:

I. The apparent flow potential is situated at the ends of the platinum capillary. If the capillary is turned over in the flow potential cell, the E. M. F. then should obtain the opposite sign. Experimentally this appeared indeed to be the case.

II. If we make the capillary wall into an unpolarizable electrode, no E. M. F. is observed in the liquid flow. We actually find this in the case that the flow is brought about with 10^{-4} n HCl + 0.1 % quinhydrone.

III. It must be possible to imitate the phenomena on the platinum capillary by means of the following arrangement: Flow potential cell with glass capillary and in each vessel, by the side of the measuring-electrode, yet a platinum plate. When they are connected by a copper wire, the same discharge and polarization phenomena must occur on these plates as before on the platinum capillary. The two arrangements appear indeed to yield corresponding results.

IV. The potential is not constant because, owing to the continuous discharge of ions, during the pressure, the amount of gas bound by the electrode and consequently the potential differences of the hydrogen- and oxygen-electrodes change continuously.

V. After elimination of the difference in pressure, the oxygen-hydrogen cell is discharged through the content of the capillary. After a measurement, therefore, the E. M. F. decreases slowly.

VI. The electrode-potential is for the platinum capillary completely determined by the potential differences of the hydrogen- and oxygen-electrode with the liquid. According to the more or less far proceeded discharge of the cell, the electrode-potential is smaller or larger.

When we perform a flow measurement on a platinum capillary, then the amount of gas bound by the electrodes, the ε potential and consequently ζ are altered. If we keep that amount constant, a condition for a sharply defined ε and ζ, no polarization phenomena can occur.

From this follows the fundamental impossibility to determine the electrokinetic potential of platinum accurately by means of flow potential measurements.

Similarly in the determination of the isoelectric point with the multivalent ions $Al^{\cdots}$ and $Th^{\cdots\cdot}$ the amount of gas bound by the capillary plays a predominant part. Since it appears to be very difficult to reproduce this gas amount exactly, ζ also is subject to alterations and consequently the electrolyte concentration at which reversion of charge takes place. At the same time care should be taken that the amount of adsorbed gas on the capillary is in equilibrium with the amount of oxygen in the solutions which are used.

Testing of the law of VON SMOLUCHOWSKI [2]) was done as follows: The platinum capillary was replaced by a Jena glass filter 7N4. We imitated the wall conduction with two platinized platinum plates on either side of the filter, between which was a resistance box. The platinum plates were made into unpolarizable electrodes by taking 10^{-4} n HCl + 0.1 % quinhydrone as filling. With this arrangement the formula of VON SMOLUCHOWSKI appears to be applicable, within the experimental error (1 %), to an area of "wall conduction" of 400—100.000 Ω.

Utrecht, April 1937.

VAN 'T HOFF-*Laboratory.*

Chemistry. — *On the Possibility of Distinguishing Right- and Left-handed Structures in Crystals by means of their* LAUE-Patterns. By J. TER BERG and F. M. JAEGER.

(Communicated at the meeting of April 24, 1937).

§ 1. In the chemical and crystallographical literature of to-day often still the opinion is held that, — according to FRIEDEL's principle [1]) of the superimposing of a centre of symmetry on the true symmetry of the crystal, when the latter is radiated through by *X*-rays, — it is in general impossible to distinguish the dextro- and the levogyratory crystals of optically active substances from each other by means of the LAUE-patterns obtained with them.

As a general rule this, of course, is true; however, in some particular cases there are exceptions to this rule and it is possible, by a suitable arrangement of the experimental conditions, yet to distinguish the right- and left-handed structures in this way. A general review of the problem, therefore, seems to be not out of place and is the aim of the present paper.

From FRIEDEL's principle the particular symmetry of the LAUE-pattern, obtained by means of a pencil of *X*-rays penetrating a crystal in a direction parallel to an axis of symmetry of the crystal, can in each case be predicted, if the proper symmetry of the crystal-species investigated and

[1]) G. FRIEDEL, Compt. rend. Paris, **157**, 1533, (1913).

the orientation of the photographic plate and the crystal with respect to the primary X-ray are accurately known. The results of the study of such LAUE-patterns have, — in all cases where the mutual orientation of plate and crystal were, indeed, faultless, — fully corroborated the exactness of those theoretical deductions [1]). Thus, for instance, it was stated, that the special distribution of the intensities of the diffraction-spots in the LAUE-patterns of uniaxial crystals, obtained by radiating through plan-parallel plates cut parallel to {001} or {0001} of such crystals, manifested the lack of symmetry with respect to definite planes perpendicular to the photographic film, if only the crystal did *not* possess either such planes of symmetry, or symmetry-axes of *even* period perpendicular to the direction of the principal binary, ternary, quaternary or senary axis. For this reason *dolomite* and *phenakite* [symmetry *): $A_3 \,.\, C$ (also $\bar{A}_6$)]; *apatite* [symmetry: $A_6 \,.\, \Sigma \,.\, C$]; *nephelite* [symmetry: A_6]; *scheelite* and *erythrite* [symmetry: $A_4 \,.\, \Sigma \,.\, C$]; *wulfenite* [symmetry: A_4]; etc. all yielded LAUE-patterns on the basal face, the intensity-distribution of which proved only to have a single trigonal, hexagonal or tetragonal axis; and the same was stated in the case of *monoclinic* crystals, if radiated through in a direction perpendicular to {010}. On the other hand, a basal LAUE-pattern of $NiSO_4 + 6H_2O$ [symmetry: $A_4 \,.\, 2A_2' \,.\, 2A_2''$]; of *quarz* and *cinnabar* [symmetry: $A_3 \,.\, 3A_2$]; of *antimonylbariumtartrate* $+ KNO_3$ [symmetry: $A_6 \,.\, 3A_2' \,.\, 3A_2''$], yielded images of this kind, which proved to be symmetrical with respect to 2×2; 3; or 2×3 planes passing through the principal axis A_4, A_3 or A_6 respectively.

Optically active substances which crystallize in one of the eleven [2]) axially symmetrical classes: C_1—C_6; D_2—D_6; T and K, — will, therefore, in most cases yield LAUE-patterns which cannot be distinguished from their mirror-images, if the crystals are radiated through in a direction perpendicular to one or more symmetry-axes of even periods. More especially this holds for the crystals of the dihedral groups: D_2, D_3, D_4 and D_6, — as these all possess binary axes perpendicular to their principal axis A_n, — if they are radiated through in the direction of this principal axis. The same is true for crystals of the classes T and O, if the pencil of X-rays traverses the crystals in the direction of their binary ($\perp \{100\}$; $\{110\}$) or quaternary ($\perp \{100\}$) axes. But an exclusively axial symmetry of the LAUE-pattern with crystals of one of the eleven classes mentioned will certainly appear in all cases, where *no* symmetry-axes of even period

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[1]) F. M. JAEGER, Proceed. R. Acad. Sc. Amsterdam, **17**, 1204, (1915); H. HAGA and F. M. JAEGER, ibid., **18**, 542, 1201, 1350, (1916).

*) The symbols used are: A_n = principal axis of period $\frac{2\pi}{n}$; A_2' or A_2'' are binary axes of two alternating sets; Σ is the principal plane of symmetry, perpendicular to A_n; C = symmetry-centre.

[2]) F. M. JAEGER, *Principle of Symmetry*, 2nd Ed., (1920), 81.

are present perpendicular to the direction of the pencil of X-rays used in the experiment. Thus, for instance, such a dissymetry of the intensity-distribution of the diffraction-spots will certainly be observed with plan-parallel plates cut perpendicular to the principal axis of the classes: C_1, C_2, C_3, C_4 and C_6 and also in class D_3 for a plate perpendicular to its polary binary axes or in the classes: T and O for plates cut parallel to {111}. Although in the class K_H of the cubic system no optically active substances will crystallize, it is worth while to remark here that also in this class plates cut parallel to {111} will show this dissymmetry of their trigonally symmetrical LAUE-patterns. The facts mentioned were demonstrated in several cases; for instance, in the case of *quartz* [1]) $\perp \{\bar{1}2\bar{1}0\}$ (normal pattern, Plate IV, fig. 13), in class K_H for *ferric ammonium-*, and *chromic potassium alum* [2]) $\perp \{111\}$ (Plate IV, figs. 2 and 3): furthermore for *sodium chlorate* $\perp\{111\}$ and doubtlessly it will also be found with substances such as *cuprite, ammonium chloride,* SCHLIPPE's *salt,* and the *nitrates* of *Ba, Sr, Pb*; etc.

On the other hand, the basal LAUE-patterns of *d-* and *l-*$\{Co(Ene)_3\}Br_3 + H_2O$ proved to be identical, — as might have been expected, because here there are two sets of binary axes present perpendicular to the tetragonal axis A_4.

§ 2. Now let us consider such simple cases as, for instance, the basal images in the classes C_1—C_6 with their unique symmetry-axes (polar) or that of non-polar *apatite* [symmetry $A_6 \,.\, \Sigma \,.\, C$], radiated through in the direction of the principal axis.

If the X-ray source and the photographic plate be kept in an *unchanged* and *identical* position, but in two successive experiments the crystal is turned through 180° round a direction $\perp$ principal axis, — so that in the first experiment the one and in the second experiment the other end of the principal axis points towards the luminous source, — then the two LAUE-patterns obtained *on the same side* of the photographic film will prove to be each other's mirror-images and, — *as long as they are moved in their own plane only,* — they cannot be brought to coincidence with each other. The question as to whether the one or the other end of the principal axis points towards the X-ray focus (or towards the photographic film) is, of course, of particular interest in those cases where A_n is a *polar* axis, because now the two ends are *non-equivalent*; so that, when the morphological development of the terminal crystal-facets at both ends of the axis clearly proves to be different (*hemimorphic* habitus), or when the said non-equivalence of the two poles can in some other way (*thermo-* or *piezo-electrical* phenomena, *figures of corrosion*;

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[1]) H. HAGA and F. M. JAEGER, loco cit., 549.
[2]) Idem, **17**, 1204, (1915).

etc.) easily be stated, the fact mentioned above may be used in the case of dextro- and levogyratory substances for distinguishing between the one and the other spatial configuration. For if, for instance, the crystals of the two antipodes in the two successive experiments indicated above be placed between the *X*-ray source and the photographic film in *exactly the same* position, — i.e. in both cases either with their *analogous* or with their *antilogous* poles towards the luminous source, — then the obtained LAUE-patterns will, in each of the two cases mentioned, manifest a truly non-superimposable intensity-dissymmetry of their spot-distribution and thus appear as planar mirror-images of each other.

In this connection still another remark can be made: if the pencil of *X*-rays does not traverse the optically active crystal along a symmetry-axis, but in some direction including with it an arbitrary angle α ($< 90°$), the LAUE-pattern obtained will, of course, appear distorted and, therefore, be quite unsymmetrical. But if the experiment, under the same rigourous conditions as emphasized in the above, first be made with a crystal of the dextrogyratory component and subsequently with a crystal of the levogyratory substance, — the angle α being in the two cases kept exactly the same, — the unsymmetrical LAUE-pattern *I* also will prove to be the planar "mirror-image" of *II*. In this way, the *d* and *l*-crystals in each case can, therefore, also be distinguished from each other with certainty, — although, of course, this latter method will, in general, practically be of no great significance.

More particularly where the question may arise whether *isomorphous*, analogously constituted optically active compounds of different directions of rotation possess *analogous* or *antilogous* spatial configurations, the experiments of this kind can, without any other chemical or optical investigations, bring a final decision.

§ 3. As an example of this, we here will deal with the case of the optically active, isomorphous complex salts of the type: $\{Me^{III}(Base'')_3\}X_3$, in which *Base''* = *d-* or *l-Chxn* (= *cyclohexane-trans-1-2-diamine*) and Me^{III} are *Co*, *Cr* or *Rh*, — which metals can in these salts substitute each other in a rigorously isomorphous way. It is true, — the mutual spatial configurations of these salts can be and have been established in a number of other ways, — either by the study of their rotatory dispersion, by that of the solubilities of their derivatives with a similar optically active substance or by means of DÉLÉPINE's method of the so-called "optically active racemates" [1]). But it is of interest to show that perfectly agreeing results also can be obtained by means of a simple study of the LAUE-patterns of these salts.

The isomorphous salts under consideration crystallize with $3H_2O$

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[1]) Cf. F. M. JAEGER, these Proc., **40**, 3, 108, (1937); Nachrichten Ges. d. Wiss. Göttingen, N. F. **1**, 199, (1937); Bull. Soc. chim. franç., (5), **4**, (1937).

(*Cr*, *Rh*) or with $4H_2O$ (*Co*) and they all have quite comparable crystal-forms. They are *hexagonal-pyramidal* (class C_6) and have a typical *hemimorphic* habitus: $p = \{10\bar{1}1\}$, predominant; $\omega = \{10\bar{1}\bar{1}\}$, narrow; $c' = \{000\bar{1}\}$, large and lustrous; $c = \{0001\}$ often absent, but, when present, only very small, — much smaller than c'. The polarity of the principal axis, therefore, is here immediately recognizable and the two ends of it can in each case be readily distinguished and identified [1]).

Of the *dextro*gyratory salts, all containing 3 molecules of *l-Chxn* within their complex ions, LAUE-patterns were made in such a way that the pencil of *X*-rays (*tungsten*-anticathode; $V = 35000$ Volts) always went along the senary axis in the direction from $\{000\bar{1}\}$ towards $\{0001\}$; the distance of the crystal from the photographic film, — of which the side used was beforehand clearly marked, so that after development it could always easily be recognized, — was 5 c.M.

In this way, successively, LAUE-patterns of the *cobaltum*, the *chromium* and the *rhodium* salts were prepared; they are reproduced in the Fig. 1—3.

From these figures it is easily seen that the dissymmetry of the intensity-distribution of the spots has in all three cases exactly *the same character*, — thus proving that the spatial configuration of the molecules, as well as that of the content of the elementary cells of these crystals (6 molecules pro cell; senary screw-axis), is for all of them just *the same*.

As a consequence of the much greater diffractive power of the *Rh*-atoms in comparison with that of the *Co*- or *Cr*-atoms, the pattern in Fig. 3 shows a somewhat other aspect than those in Fig. 1 and 2; but notwithstanding this, the analogy in their character still is easily recognized, — more particularly by taking into account those diffraction-spots with the same symbols $\{h\,k\,l\}$ which show strongly deviating intensities.

Especially in cases where the crystals investigated have a polar axis, the application of this method, — if the necessary conditions of orientation be strictly fulfilled, — may be considered to be perfectly safe.

Groningen, Laboratory for Inorganic and Physical Chemistry of the University.

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[1]) F. M. JAEGER and L. BIJKERK, these Proceed., 40, 12, 116, 246, 316, (1937).

Fig. 1. LAUE-*pattern of* $D\text{-}\{Co(l\text{-}Chxn)_3\}Cl_3+4H_2O$ *perpendicular to* $\{0001\}$.

Fig. 2. LAUE-*pattern of* $D\text{-}\{Cr(l\text{-}Chxn)_3\}+3H_2O$ *perpendicular to* $\{0001\}$.

Fig. 3. LAUE-*pattern of* $D\text{-}\{Rh(l\text{-}Chxn)_3\}+3H_2O$ *perpendicular to* $\{0001\}$.

Chemistry. — *Researches on fat metabolism X* [1]. *Feeding experiments on dogs with simple saturated triglycerides.* By P. E. VERKADE, J. VAN DER LEE and A. J. S. VAN ALPHEN.

(Communicated at the meeting of April 24, 1937).

§ 1. VERKADE and VAN DER LEE [2]) have shown that administration of different saturated triglycerides, each containing only one component acid, to one and the same healthy subject under as far as possible the same conditions gives rise to the excretion of dicarboxylic acid in the urine to a very different degree. A comparative survey of the diacidogenic properties of the triglycerides investigated *with regard to the human organism* looks as follows:

tricaprylin	weakly	diacidogenic
trinonylin	fairly weakly	,,
tricaprin	strongly	,,
triundecylin	strongly	,,
trilaurin	practically not	,,
tritridecylin	not	,,

Some time ago FLASCHENTRAEGER and BERNHARD [3]) published data concerning similar experiments on dogs with sodium salts and with methyl and ethyl esters of saturated fatty acids. The drawing of conclusions from these data is hampered to some extent by the fact that these workers did not administer all the compounds of a homologous series to the same dog; it is not likely that all test-animals tended to dioic acid-acidosis and diaciduria to the same degree. Excretion of dicarboxylic acid in the urine was only noticed with the sodium salts and with esters of caproic acid, nonoic acid and capric acid; the amount of dicarboxylic acid was always considerably greater with the salt and the esters of capric acid than with the salts and esters of both other acids.

Similar experiments with sodium salts of fatty acids had already been announced in our second paper on fat metabolism [4]) and at the time of the appearance of the first of FLASCHENTRAEGER and BERNHARD's papers quoted above [3]) were in fact in progress. It has seemed to us that under the given circumstances it was best to stop our experiments in this direction. It may be stated here — in agreement with the results obtained by the above

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[1]) For tactical reasons this paper is made to precede the papers VIII and IX already announced.

[2]) Biochem. J. **28**, 31 (1934).

[3]) Z. physiol. Chem. **238**, 221 (1936); comp. also Helv. chim. acta **18**, 962 (1935).

[4]) Proc. Royal Acad. Amsterdam, **36**, 314 (1933) (Febr. 25, 1933).

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[4]) Proc. Royal Acad. Amsterdam, **36**, 314 (1933) (Febr. 25, 1933).

mentioned workers — that administration of sodium undecoate to the dog T. in an amount of about 1.3 g per kg of body weight per day for 4 days did not give rise to excretion of undecanedioic acid in the urine.

Experiments commenced at the same time on dogs with simple saturated triglycerides have, however, been continued. The results of this work are discussed in this paper.

§ 2. The healthy adult dog T. (about 12 kg, ♂), after two preceding fast-days, was given 50 g of the triglyceride to be investigated (about 4 g per kg of body weight per day) in three equal portions, always at the same time and each time as an addition to a meal which consisted generally of as far as possible equal amounts of boiled potatoes (20—30 g) and lean horse meat (80—100 g). Water was supplied as desired. The urine passed during the day of the experiment and some succeeding days was collected and tested for the presence of higher dicarboxylic acids. Data have been furnished to a sufficient extent in previous papers [5]) concerning the method of working up the urine.

An adequate interval of time was always allowed between the various experiments and during this the test-dog could resume its normal mode of life. At least two experiments were made with each of the triglycerides investigated (triheptylin-triundecylin); these always furnished qualitatively similar results. Disturbances in the experiments through vomiting, diarrhoea etc. did not occur.

§ 3. It was only after administration of *tricaprylin* that an appreciable amount of dicarboxylic acid, actually the ω-oxidation product of the component acid (C_8), was found to be present in the urine; we succeeded in isolating therefrom 0.16 g and 0.11 g of suberic acid respectively in two experiments. After administration of *trinonylin* and *tricaprin* the urine contained generally a very small amount of the corresponding ω-oxidation product (C_9 or C_{10}); some mg of azelaic acid or sebacic acid could be isolated in these cases. The experiments with *triheptylin* and *triundecylin* always gave negative results.

The results of these experiments with simple triglycerides thus fit perfectly with those of the above described experiments of FLASCHENTRAEGER and BERNHARD with the corresponding sodium salts and esters. For this reason we have considered it unnecessary to repeat the feeding experiments with triglycerides with other dogs. We believe we can say with certainty that a comparative survey of the diacidogenic properties of the triglycerides investigated *with respect to the dog* must look as follows:

triheptylin	not	diacidogenic
tricaprylin	weakly	,,
trinonylin	practically not	,,
trcaprin	practically not	,,
triundecylin	not	,,

The difference between this table and that given at the beginning of this paper is indeed very striking; the variation of the diacidogenic properties

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[5]) Comp. VERKADE and VAN DER LEE, Z. physiol. Chem. **227**, 215 (1934); VERKADE, VAN DER LEE, VAN ALPHEN and ELZAS, Proc. Royal Acad. Amsterdam, **38**, 945 (1935).

of the simple saturated triglycerides with rise in the homologous series is quite different with man than with the dog. It would serve no object to theorize on the cause of this difference here.

From our experiments and those of FLASCHENTRAEGER and BERNHARD it may indeed be concluded that the dog has little tendency to dioic acid-acidosis and diaciduria. One may be reminded here that it is very difficult to produce ketonuria in healthy dogs [6]). In our opinion the two phenomena are related and to be expected with the carnivors in general. We are here touching the connection between carbohydrate metabolism and fat metabolism, which appears in a new light after our discovery of the degradation of fatty acids by ω-oxidation and subsequent bilateral β-oxidation and will be discussed in later communications from our laboratory.

The fact that the results of our experiments with simple triglycerides agree qualitatively completely with those of the experiments carried out by FLASCHENTRAEGER and BERNHARD [3]) with the sodium salts of the corresponding fatty acids, is a proof of the view that the observed differences in the diacidogenic properties of the triglycerides are not caused, or at least not to an appreciable extent, by differences in the velocity with which these substances are hydrolysed in the intestine. Although we have never postulated anything of the kind, one ought a priori to take this possibility into account. It is for this reason, among others, that comparative investigations have been made in our laboratory on the velocity of saponification of a series of simple triglycerides under the influence of pancreatic lipase; we will here merely refer to this work which has only been published in part as yet [7]).

This parallelism in the behaviour of the triglycerides and the corresponding sodium salts is obviously likewise contradictory to the hypothesis, put forward at the time by FLASCHENTRAEGER, BERNHARD, LOEWENBERG and SCHLAEPFER [8]) that ω-oxidation only occurs with glycerides resorbed as such, *i.e.* without preceding saponification. We need not go into the matter here as this opinion is not maintained in latter communications from FLASCHENTRAEGER and his co-workers.

We wish to thank the VAN 'T HOFF-Fund and the HOOGEWERFF-Fund for assistance in carrying out this work.

Rotterdam, March 1937.

Laboratory of the Dutch Commercial University.

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[6]) Comp. *e.g.* VON NOORDEN and ISAAC, Die Zuckerkrankheit und ihre Behandlung, 8th impr., 1927, p. 184.

[7]) HOLWERDA, VERKADE and DE WILLIGEN, Rec. trav. chim. **55**, 43 (1936); **56**. 382 (1937).

[8]) Z. physiol. Chem. **225**, 157 (1934).

Geology. — *Ueber metamorphe Gesteine am Torne Träsk (Lappland).* By H. A. BROUWER.

(Communicated at the meeting of April 24, 1937).

In den meist flachliegenden Gesteinen am Torne Träsk nimmt im allgemeinen die Metamorphose von Osten nach Westen zu, wobei weniger metamorphe Gesteine von stärker metamorphen Gesteinen überlagert werden [1]). Das kaum veränderte östliche Kambrosilur ist von kataklastischen Gesteinen überschoben. Darauf folgen harte Schiefer, die von den vielfach granat- und biotitreichen kristallinen Hochgebirgsbildungen überlagert werden. Aber auch im westlichen Teil kommen kaum veränderte klastische Gesteine zwischen den Hochgebirgsbildungen und dem Grundgebirge vor. Die Untersuchung von Gesteinen, die von mir während einer Reise im Sommer 1936 gesammelt wurden, hat zu den nachfolgenden Ergebnissen geführt.

Retrograde Metamorphose am Nuolja.

Am östlichen Abhang des Nuolja wurden im Nuoljabäcken bis hinauf zum Wasserfall Gesteine gesammelt, die zum Teil eine deutliche retrograde Metamorphose zeigen.

Am oberen Teil des Wasserfalls stehen granatreiche und granatfreie Glimmerschiefer und auch Kalkglimmerschiefer an. Weiter unten folgt ein Kalksteinhorizont, und noch tiefer folgen Gesteine, die vielfach schon makroskopisch durch ihr matteres Aussehen auffallen und in denen mikroskopisch eine retrograde Metamorphose deutlich erkennbar ist.

Die Granate in den granatreichen Glimmerschiefern am oberen Teil des Wasserfalls sind reich an Quarzeinschlüssen, deren Anordnung zeigt, dass die Kristalle im letzten Stadium ihres Wachstums in vielen Fällen stark gedreht wurden. Der Granat zeigt keine Umwandlungserscheinungen. Auch die Biotite sind zum Teil reich an Einschlüssen; Druck- und Umwandlungserscheinungen sind kaum entwickelt. Diese grösseren Kristalle liegen in einer hauptsächlich aus Quarz und farblosem Glimmer aufgebauten Grundmasse, in der auch Chlorit, Turmalin, Erz und Apatit vorkommen.

Die untersuchten granatfreien Glimmerschiefer sind reich an Graphit; eine deutliche Kleinfältelung mit Schieferung durch Umfaltung geht ungestört durch die Biotitporphyroblasten hindurch. Die Gesteine sind also

[1]) P. J. HOLMQUIST: Die Hochgebirgsbildungen am Torne Träsk in Lappland, Exkursionsführer No. 6, Congrès Géol. Intern. Stockholm 1910, S. 10.

prä- bis parakristallin gefaltet. Die Biotite zeigen zum Teil eine lamellare Verwachsung mit einem amesitreichen Chlorit. Die Gesteine enthalten zahlreiche grosse, optisch anomale, zweiachsige Apatite. Druckerscheinungen mit undulöser Auslöschung, wie sie in den später zu beschreibenden retrometamorphen Gesteinen entwickelt sind, kommen in diesen Gesteinen und auch in den hier gesammelten Kalkglimmerschiefern kaum vor.

Deutliche Erscheinungen von retrograder Metamorphose zeigen einige Gesteine, die unterhalb des Kalksteinhorizonts gesammelt wurden. Die untersuchten granatreichen Glimmerschiefer unterscheiden sich von denen am oberen Ende des Wasserfalls durch ein matteres Aussehen und eine mehr grünliche Farbe. Mikroskopisch sind die Granate zum grössten Teil in Chlorit umgewandelt; manche Kristalle zeigen eine ausgezogene Form. In einigen Gesteinen wurde beobachtet, dass der Granat in kleine Stücke aufgelöst ist, die zusammen mit Chlorit und Serizit auf den Schistositätsflächen vorkommen und andeuten, wie ursprüngliche Granate durch die jüngere retrograde Metamorphose ganz verschwunden sein könnten. Der Biotit ist im allgemeinen weniger stark umgewandelt wie der Granat. Er liegt, wie auch in den anderen Gesteinen am Nuolja, meist schräg zur Schieferung. Die undulös auslöschenden, verbogenen und gerundeten Kristalle sind an den Enden ausgezogen und tragen Schwänze von Chlorit, die sich oft weit vom Biotitkristall auf den Schieferungsflächen fortsetzen.

In graphitreichen Granatglimmerschiefern ist der Granat meist ausgezogen und ganz umgewandelt. Dasselbe gilt in geringerem Masze auch für den Biotit.

Durch detaillierte Untersuchungen muss die Abwechslung der verschiedenen Gesteinstypen näher festgestellt werden. Unsere Beobachtungen deuten aber darauf hin, dass während der Bildung der Ueberschiebungen im Granatglimmerschiefer-Komplex kaledonisch mobilisierte Zonen neben Schichtpaketen, die mehr „en bloc" transportiert wurden, vorkommen. In den mobilisierten Zonen wurde die retrograde Metamorphose gefördert.

Dunkle graphitreiche Schiefer sind zum Teil granatfrei, zum Teil reich an umgewandeltem Granat, und granatfreie dunkle Phyllite könnten durch retrograde Metamorphose aus granatreichen Glimmerschiefern hervorgegangen sein. Einlagerungen von schwarzen Phylliten sind nach HOLMQUIST [1]) für die Granatglimmerschiefer am Torne Träsk sehr charakteristisch und für die Tektonik des Granatglimmerschiefer-Phyllit-Komplexes ist eine detaillierte Untersuchung der retrograd metamorphen Erscheinungen also von grosser Bedeutung, um zu entscheiden, wie die Wechsellagerung entstanden ist. Der verschiedene Charakter der Gesteine kann auch durch die wechselnde Zusammensetzung des Ausgangsmaterials beeinflusst sein. Vielfach werden die dunklen Phyllite und die Granatglimmerschiefer auch als Gesteine verschiedenen Alters betrachtet. Am Nuolja kommen granatfreie, schwarze Phyllite vor, an denen keine Retrometamorphose

[1]) P. J. HOLMQUIST. loc. cit. S. 45.

festgestellt werden konnte. Die oben erwähnten dunklen Schiefer mit umgewandelten und ausgezogenen Granaten sind aber den schwarzen Phylliten makroskopisch schon ähnlich und nähere Untersuchungen müssen zeigen ob und inwieweit durch retrograde Metamorphose ein ursprünglicher Altersunterschied vorgetäuscht wird.

Die Hartschiefer im Abiskojokk.

In dieser der vorigen unterlagernden Gesteinsserie konnte eine retrograde Metamorphose nicht mit Sicherheit festgestellt werden, während QUENSEL [1]) für die Hartschiefer des Kebnekaisegebiets zu dem Resultat kommt, dass sie dort, wenigstens zum Teil, aus eruptiven Gesteinen durch Ultramylonitisierung hervorgegangen sind. Im Abiskojokk, wo Dolomite und Quarzite in den Hartschiefern eingelagert sind, haben Gesteine vom Charakter metamorpher Sedimente eine grosse Verbreitung. Aber besonders die Quarz-Feldspat-Einlagerungen, die in den Hartschiefern des Abiskojokk vorkommen, könnten auch aus (injizierten?) Eruptivgesteinen hervorgegangen sein und schon HOLMQUIST [2]) hat diese Einlagerungen vom Gesichtspunkte einer sedimentären Entstehung aus als schwer verständlich betrachtet.

Die Einlagerungen, die stromaufwärts von der Brücke des Weges nach Silverfallet vorkommen, bestehen aus grossen Kristallen von Kalifeldspat, die zum Teil die Gitterstruktur des Mikroklins zeigen und vielfach gespalten sind. Die Klüfte, die hauptsächlich mehr oder weniger senkrecht zur Schieferung der Hartschiefer stehen, sind mit einem Quarzgemenge ausgefüllt, das mit dem Quarzgemenge der hellen Bänder des Hartschiefers übereinstimmt und damit zusammenhängt. Biotit ist in kleinen neugebildeten Kristallen vorhanden. Das mikroskopische Bild der Einlagerungen ist durch Uebergänge mit dem des benachbarten Hartschiefers verbunden, der durch tektonische Umformung aus ähnlichem — vielleicht z. T. feinkörnigerem — Material wie dem der Einlagerungen hervorgegangen ist.

Die Hartschiefer zeigen, auch dann, wenn sie nicht schon makroskopisch deutlich gebändert sind, im allgemeinen eine Abwechslung von heller und dunkler gefärbten Zonen. Die dunkler gefärbten Zonen sind sehr reich an einem meist schwach grünlichen Glimmer, dem bräunlicher Glimmer beigemischt sein kann. Quarz und Feldspat können diesen Zonen ganz fehlen. Die helleren Zonen bestehen hauptsächlich aus einem Quarz-Feldspatgemenge, in dem öfters kleine Biotitkriställchen vorkommen. Der Feldspat ist hauptsächlich Kalifeldspat, bei Fehlen des Feldspats werden die helleren Zonen rein quarzitisch. Auch Turmalin, Rutil, Mineralien der Epidotgruppe und Erz kommen in den Gesteinen vor. Ein Karbonatgehalt ist vielleicht auf die Nähe der Karbonatsedimente zurückzuführen. Wenn

[1]) P. QUENSEL. Zur Kenntnis der Mylonitbildung, erläutert an Material aus dem Kebnekaisegebiet. Bull. Geol. Inst. of Upsala. Vol. XV, 1916, S. 91-116.

[2]) P. J. HOLMQUIST, loc. cit. S. 39.

die Quarz-Feldspat-Einlagerungen feldspatreiche Sedimente wären, dann könnten die Hartschiefer des Abiskojokk wohl ganz als eine metamorphe Sedimentserie gedeutet werden. Die mikroskopische Untersuchung schliesst diese Möglichkeit nicht aus; die quantitativen chemischen Analysen lassen verschiedene Deutungen zu, weil die während der Metamorphose stattfindenden chemischen Veränderungen nicht genau abgeschätzt werden können.

Die Analysen einer Quarz-Feldspat-Einlagerung (I) und des angrenzenden Hartschiefers (II) sind in untenstehender Tabelle zusammengestellt.

	I	II
SiO_2	75.50	66.01
TiO_2	0.43	0.84
Al_2O_3	11.22	16.83
Fe_2O_3	0.66	0.95
FeO	1.64	3.49
MnO	0.05	0.05
MgO	0.47	1.50
CaO	0.33	0.27
Na_2O	0.57	1.77
K_2O	8 79	6.68
P_2O_5	0.20	0.29
H_2O^+	0.32	1.31
H_2O^-	0.03	0.10
CO_2	—	sp.
	100.21	100.09

I, Quarz-Feldspat-Einlagerung, Abiskojokk, stromauf von der Brücke des Weges nach Silverfallet. Anal. E. F. BOUMAN.

II. Hartschiefer, direkt an die Einlagerung I grenzend, Anal. E. F. BOUMAN.

Die Voraussetzung, dass der Hartschiefer als ausgewalzter Verschiebungsmylonit, der die zugerundeten noch nicht verschieferten Einlagerungen umfliesst, gebildet wurde, wird von den Aenderungen im Chemismus nicht widersprochen. Die Abnahme des K-Gehalts könnte aus dem Fehlen von Plagioklas im Ausgangsmaterial erklärt werden, während sonst durch Abfuhr von mehr Na aus den leichter zersetzbaren Plagioklasen der K-Gehalt relativ zunimmt. Die bedeutende Abfuhr von SiO_2 kann durch eine reichliche Glimmerbildung aus Feldspat erklärt werden, unter Abfuhr des leicht beweglichen Quarzes, der auch noch den SiO_2-Gehalt der Einlagerungen erhöht haben kann.

Die Analyse II zeigt durch den Ueberschuss von Al gegenüber Ca und Alkalien und den hohen K-Gehalt die Kennzeichen eines sedimentären

Gesteins. Weil bei der Mylonitisierung granitischer Gesteine im allgemeinen die Zusammensetzung der Mylonite sich mehr derjenigen von Sedimenten nähert, wäre ein Eruptivgestein als Ausgangsmaterial nicht ausgeschlossen.

Die Analyse I zeigt durch das Verhältnis von Ca und Alkalien gegenüber Al keine Abweichung von normalen granitischen Gesteinen; nur der K-Gehalt ist hoch. Die Berechnung der normativen Mineralbestandteile nach dem amerikanischen System zeigt die Verwandtschaft mit einigen in WASHINGTON's Tabellen angegebenen Gang- und Effusivgesteinen. Die Einlagerungen können aber auch nach ihrer chemischen Zusammensetzung als ursprünglich sedimentäre Gesteine aufgefasst werden, die groberkörnige Schichten in mehr feinkörnigen Sedimenten gebildet haben können.

Die mikroskopische und chemische Untersuchung gestattet also keine sicheren Schlussfolgerungen in bezug auf einen eruptiven oder sedimentären Ursprung der Quarz-Feldspat-Einlagerungen und der benachbarten Hartschiefer. QUENSEL's Beobachtungen im Kebnekaise-Gebiet weisen auf Eruptivgesteine als Ausgangsmaterial der Hartschiefer, obwohl von ihm nicht verneint wird, dass auch ursprüngliche Sedimente darunter vorhanden sein können. Die Verhältnisse im Abiskojokk weisen auf einen sedimentären Ursprung, aber auch ursprüngliche Eruptivgesteine können anwesend sein. Jedenfalls darf wohl angenommen werden, dass die als Hartschiefer zusammengefassten Gesteine einen metamorphen Gesteinstypus darstellen, der unter den Verhältnissen der Epi-bis Mesozone sowohl aus Sedimenten als aus Eruptivgesteinen gebildet werden kann. Wie bei den Gneisen könnte man zwischen Ortho- und Parahartschiefern unterscheiden.

Bewegung und Kristallisation in den Hartschiefern.

Die Wechsellagerung von Dolomit und Hartschiefer kann zum Teil noch mit der mehr oder weniger deformierten ursprünglichen Schichtung übereinstimmen, aber das sehr wechselnde Verhalten der Dolomite, welche zu dickeren Paketen angehäuft sind, mit geringer und wechselnder Mächtigkeit kaum gefaltet mit den Hartschiefern wechsellagern, mit den Hartschiefern sehr stark verfaltet sind oder ganz auskeilen, deutet darauf hin, dass die Struktur durch starke Bewegung mit Laminierung, Faltung und Stauung der Schichten zustande gekommen ist.

Die Figur I zeigt die Wechsellagerung von Dolomit und Hartschiefer wie sie im Abiskojokk nördlich von der Brücke des Weges nach Silverfallet aufgeschlossen ist. Dasselbe Bild wiederholt sich in verschiedenen Dimensionen. Dolomiteinlagerungen erreichen eine Mächtigkeit von mehreren Metern; sie keilen zwischen den Hartschiefern aus, wie bei der Brücke des Weges nach Kärsavaggestugan, wo sie sehr stark in liegenden Falten mit den Hartschiefern verfaltet sind.

Kleinere Falten sind besonders in den dünngebänderten Hartschiefern mit starkem Farbenwechsel der Falten auffallend. Es sind hauptsächlich

liegende Falten mit gestreckten und vielfach zerrissenen und auslaminierten Mittelschenkeln, die zu Verschiebungsflächen geworden sind. Eine spitz laminierte Faltung, die auf Bewegung unter höherer Belastung deutet, ist

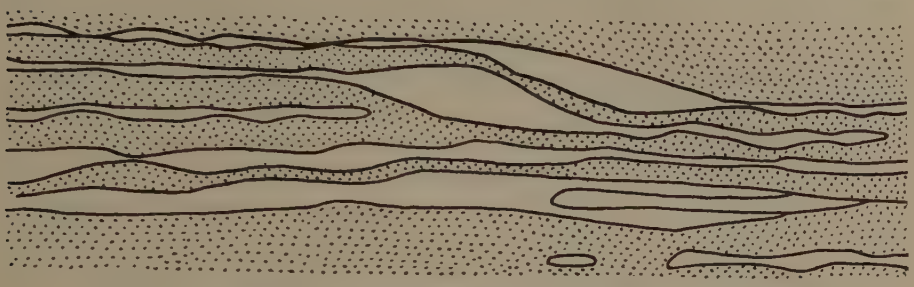

Fig. 1. Wechsellagerung von Dolomit und Hartschiefer, weiss: Dolomit punktiert: Hartschiefer. × 3/4.

verbreitet. Die Faltung wiederholt sich mit ähnlichen Kennzeichen in verschiedenen Dimensionen in demselben Gesteinskomplex.

Die Schieferung verläuft ungefähr parallel mit der Achsenebene der liegenden Falten. Besonders, wo die Hartschiefer mit den Dolomiten verfaltet sind, ist an den Umbiegungsstellen durch die schiefernde Bewegung manchmal eine Rippung entstanden, während an den Faltenschenkeln die Schieferung mehr oder weniger parallel mit der Grenzfläche zwischen Dolomit und Hartschiefer verläuft. Die Schieferung hat sich zu Lamination gesteigert, aber oft zeigt eine erkennbare Streifung auf den Trennungsflächen der nach der Schieferung in Platten trennbaren Hartschiefer den Zusammenhang mit Faltung. Wenn die Bänderung und die Schieferung genau parallel verlaufen, wird eine noch erhaltene ursprüngliche Schichtung mit Schichtfugen vorgetäuscht.

Gegen die Annahme, dass die Ausbildung der Bänderung und die Kristallisation zeitlich getrennte Erscheinungen sind, spricht auch schon das Nebeneinander-Vorkommen von deformiertem Feldspat und blastischem Quarz. In den Quarz-Feldspat-Einlagerungen werden die Klüfte in den zerbrochenen Feldspaten mit einem Quarzgemenge ausgefüllt, das durch Stoffwanderung zugeführt sein muss und mit dem Quarz der hellen Bänder der benachbarten Hartschiefer zusammenhängt und gleicher Entstehung ist. Undulöse Auslöschungen dieses blastischen Quarzes kommen vor und zeigen dann, dass nach der Zerbrechung der Feldspate und der Kristallisation des Quarzes noch Bewegung stattgefunden hat. In den stark gefalteten gebänderten Hartschiefern ist besonders an den Biotitblättchen der hellen Bänder oft deutlich erkennbar, dass sie mehr oder weniger parallel der Achsenebene gerichtet sind.

Nach dem Obengesagten glauben wir, dass Bänderung, Faltung, Schieferung und Kristallisation der Hartschiefer während derselben Deformationsperiode unter Verhältnissen ungefähr an der Grenze von Epi- und Mesozone stattgefunden haben. Bei seinen Untersuchungen über Mylonit-

bildung im Kebnekaise-Gebiet hat QUENSEL [1]) darauf hingewiesen, dass — auch wenn die Umkristallisation über die Kataklase überwiegt — zuerst die mechanische Zertrümmerung sich vollzogen hat und dass erst später Umkristallisationserscheinungen diese Struktur mehr oder weniger verhüllt haben. Es ist verständlich, dass in seinen Orthohartschiefern erst die Verkleinerung des Korns und die Ausbildung der Schieferung die Zirkulation der Lösungen erleichtern musste, bevor die Umkristallisation richtig vor sich gehen konnte. In den untersuchten Hartschiefern vom Abiskojokk sind kataklastische Erscheinungen verhältnismässig selten. In den grobkörnigen Quarz-Feldspat-Einlagerungen sind sie deutlich entwickelt. Es ist möglich, dass durch ein ursprünglich feineres Korn und eine ursprüngliche Schichtung die Umkristallisation beschleunigt wurde und in vielen Gesteinen die kataklastischen Stadien in geringerem Masz als im Kebnekaise-Gebiet oder gar nicht durchlaufen wurden.

Zusammenfassung.

Wie in den östlichen Randgebieten kommen auch im westlichen Teil des Gebirges südlich vom Torne Träsk rein klastische Gesteine — vorwiegend Arkosen und Schiefer — zwischen den Hochgebirgsbildungen und dem Grundgebirge eingelagert vor [2]). Die epi-bis mesometamorphen Hartschiefer liegen höher als die rein klastischen Gesteine. Bewegung und Kristallisation weisen auf eine kaledonische Dynamometamorphose, die während der Bildung der Ueberschiebungen in Zonen starker Bewegung stattfand. Die Kennzeichen der Hartschiefer weisen auf eine hauptsächlich parakristalline Deformation von einer z. T. aus Sandsteinen und Arkosen bestehenden Schichtserie, und es ist möglich, dass vom Anfang der Metamorphose an die Kristallisationsgeschwindigkeit in Hauptsache gleich oder grösser war als die Deformationsgeschwindigkeit. Ein einigermassen bedeutender Altersunterschied zwischen den sedimentären Hartschiefern und den rein klastischen Sedimenten kann aus dem metamorphen Charakter der von uns untersuchten Hartschiefer nicht abgeleitet werden.

In einem Teil der, durch prä- bis parakristalline Deformation gekennzeichneten, überschobenen Glimmerschiefer am Ostabhang des Nuolja kann eine ältere und eine jüngere Metamorphose unterschieden werden. Die jüngere ist — wie die der Hartschiefer — eine kaledonische Dynamometamorphose. Sie hat hier destruktiv gewirkt und hat manchen Gesteinen ein „jüngeres" Aussehen verliehen. Die ältere Metamorphose kann in den von uns untersuchten Gesteinen zeitlich nicht genau festgelegt werden; sie kann kaledonisch oder älter sein. In vielen Gebirgen sind die metamorphen Gesteine durch petrographische Vergleiche vielfach für älter

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1) P. QUENSEL. loc. cit. S. 105.
2) P. J. HOLMQUIST. loc. cit. S. 60.

gehalten als sie in Wirklichkeit sind. Sie sind allmählich in höhere stratigraphische Horizonte hinaufgerückt. Aber umgekehrt können Gesteine durch retrograde Metamorphose jüngeren Gesteinen petrographisch ähnlich werden. Die genaue Abschätzung des proportionalen Anteils der älteren konstruktiven und der jüngeren destruktiven Metamorphose in den Glimmerschiefern und Phylliten des Nuolja ist für die Tektonik dieses Gebiets von grosser Bedeutung.

Mathematics. — *Arithmetische Eigenschaften einer Klasse von Dezimalbrüchen* von KURT MAHLER in Krefeld. (Communicated by Prof. J. G. VAN DER CORPUT).

(Communicated at the meeting of April 24, 1937).

Da über die Dezimalbruchentwicklung der klassischen transzendenten Zahlen, z.B. e und π, auch heute noch fast nichts bekannt ist [1]), so hat es ein gewisses Interesse, spezielle Dezimalbrüche zu konstruieren, deren Transzendenz sich zeigen lässt, ohne trivial zu sein. Ich behandle hier Dezimalbrüche folgender Gestalt: Unter $f(k)$ werde ein ganzwertiges nichtkonstantes Polynom verstanden, das für $k \geqq 1$ positiv ist und mit k gegen $+\infty$ strebt. Alsdann bedeute σ den Dezimalbruch, der entsteht, wenn hinter das Komma der Reihe nach nacheinander die dezimal dargestellten natürlichen Zahlen $f(1)$, $f(2)$, $f(3)$, ... hingeschrieben werden; z.B. wird also so dem Polynom $f(k) = \frac{1}{2}(k^2 + k)$ der Dezimalbruch

$$0{,}1\ 3\ 6\ 10\ 15\ 21\ 28\ 36\ 45\ 55\ 66\ 78\ 91\ 105\ 120.....$$

zugeordnet. Für alle Zahlen σ dieser Gestalt wird bewiesen:

„σ ist transzendent, aber keine LIOUVILLE*-Zahl."*

Ein ganz gleichlautendes Resultat gilt auch noch, wenn statt solcher Dezimalbrüche analog gebildete Brüche in bezug auf eine beliebige Zahlsystem-Basis $q \geqq 2$ betrachtet werden; darum wird in der vorliegenden Arbeit sogleich der Fall eines allgemeinen natürlichen $q \geqq 2$ zugrunde gelegt. Jedoch mache ich die Annahme, dass $f(x)$ für alle reellen $x \geqq 1$ monoton im strengen Sinn zunimmt, weil dies die Betrachtungen wesentlich vereinfacht. Hierin liegt aber nur eine scheinbare Einschränkung; lässt man nämlich endlichviele Anfangsziffern von σ fort, was auf eine ganze lineare Transformation von σ mit rationalen Koeffizienten hinauskommt, so ist diese Forderung für die Folge der Restziffern von selbst erfüllt.

Der Beweis der beiden Aussagen über σ beruht wesentlich auf einer

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[1]) Wegen einiger elementarer Aussagen vergl. eine demnächst in Mathematica *B* (Zutphen) erscheinende Note des Verfassers. Dort wird auch schon das Ergebnis dieser Arbeit im Spezialfall $f(x) = x$ gezeigt.

auch an sich merkwürdigen Reihenentwicklung (A) für diese Zahl. Indem man diese Reihe nach beliebig vielen Gliedern abbricht, erhält man eine Folge von Brüchen, die sehr schnell gegen σ konvergieren. Die Nenner dieser Brüche sind bis auf einen Faktor geringerer Grössenordnung reine Potenzen von q; mittels eines wichtigen neuen Satzes von SCHNEIDER kann hieraus die Transzendenz von σ hergeleitet werden. Dass σ keine LIOUVILLE-Zahl ist, ergibt sich schliesslich direkt aus einem Irrationalitätsmass (B) für σ, das leicht aus den Eigenschaften seiner Näherungsbrüche folgt.

1. Sei $f(x)$ ein ganzwertiges Polynom in x genau vom Grad $m \geqq 1$, das für alle $x \geqq 1$ selbst $\geqq 1$ ist und monoton im strengen Sinn zunimmt. Die Umkehrfunktion $x = g(y)$ von $y = f(x)$ nimmt folglich für $y \geqq f(1)$ ebenfalls monoton im strengen Sinn zu und ist stets $\geqq 1$.

Bedeutet $q \geqq 2$ eine feste natürliche Zahl, so gestattet jede der natürlichen Zahlen $f(k)$ $(k = 1, 2, 3, \ldots)$ eine eindeutig bestimmte Darstellung

$$f(k) = \sum_{\lambda=0}^{N_k} Z_{k\lambda}\, q^{N_k-\lambda} = Z_{k0}\, Z_{k1}\, Z_{k2} \ldots Z_{k\,N_k}$$

im Zahlsystem zur Basis q; dabei gehören die Ziffern $Z_{k0}, Z_{k1}, \ldots, Z_{k\,N_k}$ der endlichen Folge 0,1, ..., q—1 an und es ist speziell $Z_{k0} > 0$. Indem die einzelnen Ziffern der Darstellungen aller $f(k)$ der Reihe nach hinter dem Komma niedergeschrieben werden, ergibt sich der Bruch

$$\sigma = 0, Z_{10}\, Z_{11} \ldots Z_{1\,N_1}\, Z_{20}\, Z_{21} \ldots Z_{2\,N_2}\, Z_{30}\, Z_{31} \ldots Z_{3\,N_3} \ldots$$

zur Basis q, dessen arithmetische Eigenschaften im folgenden untersucht werden sollen. Zu diesem Zweck werden wir zunächst eine einfachere Reihenentwicklung für σ herleiten.

2. Sei n die durch die Ungleichungen

$$q^{n-1} \leqq f(1) \leqq q^n - 1$$

definierte natürliche Zahl, ferner

$$j_{n-1} = 0,\ j_\nu = [g(q^\nu - 1)] \text{ für } \nu = n, n+1, n+2, \ldots.$$

Für jede natürliche Zahl $\nu \geqq n$ werde unter J_ν die Menge aller natürlichen Zahlen k mit

$$q^{\nu-1} \leqq f(k) \leqq q^\nu - 1$$

verstanden. Man sieht leicht ein, dass k dann und nur dann zu J_ν gehört, wenn

$$j_{\nu-1} + 1 \leqq k \leqq j_\nu$$

ist; J_ν enthält also $j_\nu - j_{\nu-1}$ Elemente. Da $f(k)$ im System zur Basis q dann und nur dann ν-stellig wird, wenn k in J_ν liegt, so ist demnach die Gesamtanzahl der Ziffern aller ν-stelligen Zahlen $f(k)$ gleich

$$\nu\,(j_\nu - j_{\nu-1})$$

und folglich die Gesamtanzahl der Ziffern aller höchstens $(\nu-1)$-stelligen Zahlen $f(k)$ für $\nu=n$ gleich 0 und für $\nu>n$ gleich

$$\sum_{\mu=n}^{\nu-1} \mu\,(j_\mu - j_{\mu-1}).$$

Ferner entsteht für $\nu \geqq n$ die kleinste, bzw. die grösste ν-stellige Zahl $f(k)$, wenn $k=j_{\nu-1}+1$, bezw. $k=j_\nu$ ist.

Somit ist der additive Beitrag, den die Ziffern von $f(k)$ zu σ liefern, für k in J_n gleich

$$f(k)\,q^{-nk}$$

und für k in J_ν mit $\nu>n$ gleich

$$f(k)\,q^{-\sum_{\mu=n}^{\nu-1}\mu(j_\mu-j_{\mu-1})-\nu(k-j_{\nu-1})},$$

denn eine etwaige λ-te Stelle Z hinter dem Komma in der Entwicklung von σ zur Basis q liefert den Beitrag $Zq^{-\lambda}$ zum Wert von σ. Indem die zu allen $k=1, 2, 3, \ldots$ gehörigen Teilbeiträge addiert werden, ergibt sich damit für σ die folgende Reihenentwicklung:

$$\sigma=\sum_{k=1}^{j_n} f(k)\,q^{-nk}+\sum_{\nu=n+1}^{\infty} q^{-\sum_{\mu=n}^{\nu-1}\mu(j_\mu-j_{\mu-1})+\nu j_{\nu-1}} \sum_{k=j_{\nu-1}+1}^{j_\nu} f(k)\,q^{-\nu k}.$$

3. Diese Entwicklung kann noch vereinfacht werden, da die Summen

$$S_\nu=\sum_{k=j_{\nu-1}+1}^{j_\nu} f(k)\,q^{-\nu k} \qquad (\nu=n, n+1, n+2, \ldots$$

sich bekanntlich explizit angeben lassen. Am einfachsten gelingt dies mittels der folgenden Formel aus der Differenzenrechnung [2]):

$$\sum_{z=0}^{\infty} F(z)\,x^z=\sum_{h=0}^{\infty}\frac{x^h\,\Delta^h F(0)}{(1-x)^{h+1}}.$$

Dabei bedeutet $F(z)$ eine für $z=0, 1, 2, \ldots$ definierte Funktion und

$$\Delta^h F(z)=\sum_{i=0}^{h}\binom{h}{i}(-1)^i F(z+h-i) \qquad (\Delta^0 F(z)=F(z))$$

ihre h-te Differenz an der Stelle z $(h=0, 1, 2, \ldots)$.

Um von dieser Formel Gebrauch zu machen, werde S_ν in der Form

$$S_\nu=q^{-\nu(j_{\nu-1}+1)}\sum_{z=0}^{\infty} f(z+j_{\nu-1}+1)\,q^{-\nu z}-q^{-\nu(j_\nu+1)}\sum_{z=0}^{\infty} f(z+j_\nu+1)\,q^{-\nu z}$$

geschrieben und $x=q^{-\nu}$ gesetzt. Da die Differenzen $(m+1)$-ten und

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[2]) Siehe z.B.: CESARO-KOWALEWSKI, Elementares Lehrbuch der algebraischen Analysis und der Infinitesimalrechnung (Leipzig 1904), § 807 c. Offenbar erübrigt sich im betrachteten Fall jeder Konvergenzbeweis.

höheren Grades der beiden Polynome $f(z + j^{\nu}_{-1} + 1)$ und $f(z + j_{\nu} + 1)$ identisch in z verschwinden, so folgt alsdann

$$S_{\nu} = q^{-\nu j_{\nu-1}} A_{\nu} - q^{-\nu j_{\nu}} B_{\nu}$$

mit den Abkürzungen

$$A_{\nu} = \sum_{h=0}^{m} \frac{\Delta^h f(j_{\nu-1}+1)}{(q^{\nu}-1)^{h+1}}, \quad B_{\nu} = \sum_{h=0}^{m} \frac{\Delta^h f(j_{\nu}+1)}{(q^{\nu}-1)^{h+1}}.$$

Mittels dieser Gleichung lassen sich alle Teilsummen S_{ν} aus der im vorigen Paragraphen abgeleiteten Gleichung für σ eliminieren. Das ergibt:

$$\sigma = (A_n - q^{-nj_n} B_n) + \sum_{\nu=n+1}^{\infty} q^{-\sum_{\mu=n}^{\nu-1} \mu(j_{\mu}-j_{\mu-1})+\nu j_{\nu-1}} (q^{-\nu j_{\nu-1}} A_{\nu} - q^{-\nu j_{\nu}} B_{\nu}),$$

und schliesslich durch Zusammenfassen aller Glieder mit gleichem Exponenten von q und Einsetzen der Werte von A_{ν} und B_{ν}:

$$(A): \quad \sigma = \sum_{h=0}^{m} \frac{\Delta^h f(1)}{(q^n-1)^{h+1}} +$$

$$+ \sum_{\nu=n+1}^{\infty} q^{-\sum_{\mu=n}^{\nu-1} \mu(j_{\mu}-j_{\mu-1})} \sum_{h=0}^{m} \Delta^h f(j_{\nu-1}+1) \left\{ \frac{1}{(q^{\nu}-1)^{h+1}} - \frac{1}{(q^{\nu-1}+1)^{h+1}} \right\}.$$

4. Das letzte Ergebnis führt nun leicht zur Aufstellung einer Folge von Brüchen P_s/Q_s, die ausserordentlich schnell gegen σ konvergieren und damit den Transzendenzbeweis ermöglichen. Sei D_s für jeden Index $s \geqq n+1$ das kleinste gemeinsame Vielfache der Zahlen

$$q^n - 1, \; q^{n+1} - 1, \ldots, q^s - 1,$$

weiter

$$Q_s = D_s^{m+1} \, q^{\sum_{\mu=n}^{s-1} \mu(j_{\mu}-j_{\mu-1})}$$

und

$$P_s = Q_s \left\{ \sum_{h=0}^{m} \frac{\Delta^h f(1)}{(q^n-1)^{h+1}} + \right.$$

$$\left. + \sum_{\nu=n+1}^{s} q^{-\sum_{\mu=n}^{\nu-1} \mu(j_{\mu}-j_{\mu-1})} \sum_{h=0}^{m} \Delta^h f(j_{\nu-1}+1) \left(\frac{1}{(q^{\nu}-1)^{h+1}} - \frac{1}{(q^{\nu-1}-1)^{h+1}} \right) \right\},$$

ferner

$$R_s = \sum_{\nu=s+1}^{\infty} q^{-\sum_{\mu=n}^{\nu-1} \mu(j_{\mu}-j_{\mu-1})} \sum_{h=0}^{m} \Delta^h f(j_{\nu-1}+1) \left(\frac{1}{(q^{\nu}-1)^{h+1}} - \frac{1}{(q^{\nu-1}-1)^{h+1}} \right).$$

Es besteht also die Gleichung

$$\sigma - \frac{P_s}{Q_s} = R_s \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Q_s ist definitionsgemäss eine natürliche Zahl. Aber auch P_s ist ganz rational, denn die Koeffizienten

$$\Delta^h f(1) \quad \text{und} \quad \Delta^h f(j_{\nu-1}+1)$$

sind als Differenzen ganzwertiger Polynome an natürlichen Stellen des Arguments ganze rationale Zahlen, so dass die Nenner der einzelnen Summanden von P_s durch den Faktor Q_s weggehoben werden. Wir werden für *log* Q_s und *log* R_s asymptotische Formeln ableiten und beginnen dazu mit einer solchen Formel für

$$i_\nu = \sum_{\mu=n}^{\nu-1} \mu (j_\mu - j_{\mu-1}) = (\nu-1) j_{\nu-1} - (j_n + j_{n+1} + \ldots + j_{\nu-2}).$$

Als Polynom m-ten Grades, das für $x \to +\infty$ gegen $+\infty$ strebt, gestattet $f(x)$ eine Darstellung

$$f(x) = \alpha^{-m} x^m \left(1 + \frac{\alpha_1}{x} + \frac{\alpha_2}{x^2} + \ldots + \frac{\alpha_m}{x^m}\right),$$

wo α eine positive Konstante und $\alpha_1, \alpha_2, \ldots, \alpha_m$ gewisse reelle Zahlen sind. Die Umkehrfunktion von $y = f(x)$ wird alsdann gleich

$$x = g(y) = \alpha y^{1/m} + O(1).$$

Wegen

$$j_\mu = [g(q^\mu - 1)]$$

ist also insbesondere

$$j_\mu = \alpha q^{\mu/m} + O(1)$$

und damit

$$i_\nu = \alpha (\nu-1) q^{\frac{\nu-1}{m}} + O\left(q^{\frac{\nu-1}{m}}\right).$$

Erstens ist nun die natürliche Zahl

$$D_s < q \cdot q^2 \cdot q^3 \ldots q^s = q^{\frac{s(s+1)}{2}},$$

so dass sich aus der vorigen Formel für i_ν und der Definition von Q_s die Beziehung

$$\log Q_s \backsim \alpha (s-1) q^{\frac{s-1}{m}} \log q \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

ergibt. Hiernach ist insbesondere

$$\lim_{s\to\infty}\frac{\log Q_{s+1}}{\log Q_s}=q^{1/m} \quad \ldots\ldots\ldots \quad (3)$$

Zweitens ist für $\nu\to\infty$

$$\sum_{h=0}^{m}\Delta^h f(j_{\nu-1}+1)\left\{\frac{1}{(q^\nu-1)^{h+1}}-\frac{1}{(q^{\nu-1}-1)^{h+1}}\right\}\backsim$$

$$\backsim -f(j_{\nu-1}+1)\frac{q-1}{q^\nu}\backsim -a^{-m}\left(a\,q^{\frac{\nu-1}{m}}\right)^m\frac{q-1}{q^\nu}=-\left(1-\frac{1}{q}\right),$$

da die Terme mit $h\geqq 1$ sich gegen den mit $h=0$ vernachlässigen lassen. Der Summand von R_s mit $\nu=s+1$ hat also offenbar höhere Grössenordnung als alle folgenden, und man erhält

$$R_s\backsim -\left(1-\frac{1}{q}\right)q^{-i_{s+1}}$$

und erst recht

$$\log|R_s|\backsim -a\,s\,q^{\frac{s}{m}}\log q \quad \ldots\ldots\ldots \quad (4)$$

Hieraus folgt für alle genügend grossen s

$$R_s\neq 0 \quad \ldots\ldots\ldots \quad (5)$$

Ferner ergibt sich wegen (2) die Limesgleichung

$$\lim_{s\to\infty}\frac{\log|R_s|}{\log Q_s}=-q^{1/m}<-1 \quad \ldots\ldots\ldots \quad (6)$$

und die asymptotische Formel

$$\log|Q_sR_s|\backsim -a\,s\left(q^{\frac{1}{m}}-1\right)q^{\frac{s-1}{m}}\log q, \quad \ldots\ldots \quad (7)$$

so dass also von einem s ab

$$|Q_sR_s|<|Q_{s-1}R_{s-1}| \quad \ldots\ldots\ldots \quad (8)$$

ist.

5. Von Th. Schneider wurde vor einiger Zeit folgender Satz bewiesen: *„Zu der reellen Zahl ϑ gebe es eine Konstante $\varkappa>1$ und eine unendliche Folge von Brüchen*

$$\frac{p_1}{q_1},\frac{p_2}{q_2},\frac{p_3}{q_3},\ldots,$$

deren Nenner von einem s ab monoton zunehmen und Potenzen einer festen natürlichen Zahl q darstellen, derart dass

$$0 < \left| \vartheta - \frac{p_s}{q_s} \right| \leqq q_s^{-\varkappa} \quad \text{und} \quad \limsup_{s \to \infty} \frac{\log q_{s+1}}{\log q_s} < \infty$$

ist. Dann ist ϑ transzendent." Eine Durchsicht des Beweises dieses Satzes zeigt ohne Mühe, dass ϑ auch dann noch transzendent ist, wenn die Bedingung, dass die q_s Potenzen von q sind, durch folgende schwächere Forderung ersetzt wird [3]): *„Für jeden Index s kann $q_s = q_s' q_s''$ als Produkt einer natürlichen Zahl q_s' mit*

$$\lim_{s \to \infty} \frac{\log q_s'}{\log q_s} = 0$$

und einer reinen Potenz q_s'' von q dargestellt werden."

Identifizieren wir ϑ mit σ und nehmen wir eine willkürliche Zahl $\varkappa$ mit

$$1 < \varkappa < q^{1/m},$$

so haben die Näherungsbrüche P_s/Q_s wegen (2) von einem s ab monoton zunehmende Nenner; nach Definition ist ferner $Q_s = D_s^{m+1} \cdot q^{i_s}$, wo der erste Faktor der Gleichung

$$\lim_{s \to \infty} \frac{\log D_s^{m+1}}{\log Q_s} = 0$$

genügt und der zweite eine reine Potenz von q ist; endlich gilt wegen (1), (5) und (6) für genügend grosse s

$$0 < \left| \sigma - \frac{P_s}{Q_s} \right| < Q_s^{-\varkappa}$$

und es ist wegen (3)

$$\limsup_{s \to \infty} \frac{\log Q_{s+1}}{\log Q_s} < \infty.$$

Alle Voraussetzungen der vorigen Verallgemeinerung des SCHNEIDERschen Satzes sind demnach erfüllt, und man kommt zu folgendem Resultat:

Satz 1: *Die Zahl σ ist transzendent.*

6. Zu jedem Bruch P/Q mit schon genügend grossem Nenner Q kann wegen (8) ein Index s mit

$$|Q_s R_s| < \frac{1}{2Q} \leqq |Q_{s-1} R_{s-1}| \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

[3]) Da SCHNEIDER selbst seinen Beweis nur skizziert, so werde verwiesen nach folgender Arbeit des Verfassers: „Ein Analogon zu einem SCHNEIDERschen Satz", Proc. Royal. Akad., Amsterdam, **39**, 633—640 u. 729—739 (1936). In dem dort in allen Einzelheiten bewiesenen Satz 2 ist der Satz von SCHNEIDER als Spezialfall enthalten.

gefunden werden. Wegen der Identität

$$\sigma - \frac{P}{Q} = \left(\frac{P_s}{Q_s} - \frac{P}{Q}\right) + R_s$$

ist alsdann entweder $\frac{P_s}{Q_s} = \frac{P}{Q}$ und also

$$\sigma - \frac{P}{Q} = R_s,$$

oder $\frac{P_s}{Q_s} \neq \frac{P}{Q}$, also $|P_s Q - P Q_s| \geqq 1$ und somit wegen (9):

$$\left|\sigma - \frac{P}{Q}\right| \geqq \frac{1}{Q Q_s} - |R_s| \geqq \frac{1}{2 Q Q_s} \geqq \frac{1}{2}|R_s|,$$

so dass in jedem Fall

$$\left|\sigma - \frac{P}{Q}\right| \geqq \frac{1}{2}|R_s| \quad \ldots\ldots\ldots \quad (10)$$

folgt.

Wenn nun aber Q und also auch s genügend gross ist, so gilt wegen (9) und (7)

$$\frac{1}{Q} \leqq q^{-\frac{2}{3}\alpha_s (q^{1/m}-1)\, q^{\frac{s-2}{m}}},$$

und wegen (10) und (4)

$$\left|\sigma - \frac{P}{Q}\right| \geqq q^{-\frac{4}{3}\alpha_s q^{\frac{s}{m}}},$$

also erst recht

(*B*): $$\left|\sigma - \frac{P}{Q}\right| \geqq Q^{-\frac{2q^{2/m}}{q^{1/m}-1}}.$$

Damit ist bewiesen:

Satz 2: *Alle Näherungsbrüche von σ mit genügend grossem Nenner genügen der Ungleichung (B). Die Zahl σ ist also Nicht-Liouvillesch.*

Herrn Dr. med. A. HEILBRONN gewidmet.

Krefeld, März 1937.

Mathematics. — *Two remarks on* VAN DER CORPUT's *generalisation of* KNOPP's *inequality.* By V. LEVIN. (Communicated by J. G. VAN DER CORPUT).

(Communicated at the meeting of April 24, 1937).

1. Let $a_n \geqq 0$ (not all $a_n = 0$), $0 < u < 1$, $q > 0$, $0 < p \leqq 1$. Then

$$F(u) = \sum_{n=0}^{\infty} u^n \left(a_0^{\binom{n}{0} q^n} a_1^{\binom{n}{1} q^{n-1}} \ldots a_n^{\binom{n}{n}} \right)^{\frac{p}{(q+1)^n}} \leqq \frac{q\lambda + 1}{(1-\lambda)^{1-p}} \left\{ \sum_{n=0}^{\infty} a_n \right\}^p \quad . \quad (1)$$

where $0 < \lambda = \lambda(u, p, q) < 1$ is uniquely determined by

$$u = \frac{(q+1)\lambda^{1-\frac{p}{q+1}}}{q\lambda + 1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1,1)$$

The sign of equality in (1) holds for $a_n = c\lambda^n$ $(c > 0)$. VAN DER CORPUT's inequality [1]) is (1) with $p = 1$, in which case $u = 1$ is admitted (KNOPP's inequality). But for $p = 1$, $u = 1$ equality cannot occur in (1), the constant $q + 1$ remaining, however, the best possible.

The proof of (1) runs as follows:

$$F(u) = \sum_{n=0}^{\infty} u^n \lambda^{\frac{np}{q+1}} \left(\left(a_0^p \lambda^{-p.0} \right)^{\binom{n}{0} q^n} \left(a_1^p \lambda^{-p.1} \right)^{\binom{n}{1} q^{n-1}} \ldots \left(a_n^p \lambda^{-pn} \right)^{\binom{n}{n}} \right)^{\frac{1}{(q+1)^n}}$$

$$\leqq \sum_{n=0}^{\infty} u^n \lambda^{\frac{np}{q+1}} \frac{1}{(q+1)^n} \sum_{m=0}^{n} \binom{n}{m} q^{n-m} a_m^p \lambda^{-mp}$$

$$= \sum_{m=0}^{\infty} a_m^p \left(\frac{u\lambda^{-\frac{pq}{q+1}}}{q+1} \right)^m \sum_{n=m}^{\infty} \binom{n}{m} \left(\frac{q u \lambda^{\frac{p}{q+1}}}{q+1} \right)^{n-m}$$

$$= \sum_{m=0}^{\infty} a_m^p \left(\frac{u\lambda^{-\frac{pq}{q+1}}}{q+1} \right)^m \left(1 - \frac{q u \lambda^{\frac{p}{q+1}}}{q+1} \right)^{-m-1}$$

$$= (q\lambda + 1) \sum_{m=0}^{\infty} a_m^p \lambda^{m(1-p)}$$

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[1]) "Generalisation of an inequality by KNOPP", these Proceedings **39** (1936), 1053—1055.

by (1, 1). Hence

$$F(u) \leqq (q\lambda+1)\left\{\sum_{n=0}^{\infty} a_n\right\}^{p}\left\{\sum_{n=0}^{\infty}\lambda^n\right\}^{1-p}$$

$$= \frac{q\lambda+1}{(1-\lambda)^{1-p}}\left\{\sum_{n=0}^{\infty} a_n\right\}^{p}.$$

2. From his inequality VAN DER CORPUT [2]) deduces the following:

$$\sum_{n=0}^{\infty}\frac{1}{n+1}\left(a_0^{\binom{n}{0}}\, a_1^{\binom{n}{1}}\ldots a_n^{\binom{n}{n}}\right)^{\frac{1}{2^n}} < (\pi-2)\sum_{n=0}^{\infty} a_n,$$

where $(\pi-2)$ is not the best possible constant [3]). We prove

$$\sum_{n=0}^{\infty}\frac{1}{n+1}\left(a_0^{\binom{n}{0}q^n}\, a_1^{\binom{n}{1}q^{n-1}}\ldots a_n^{\binom{n}{n}}\right)^{\frac{1}{(q+1)^n}} < \sum_{n=0}^{\infty}\frac{n+2+q}{(n+1)(n+2)}\, a_n, \quad . \quad (2)$$

where the factor 1 of the right hand side is the best possible one. Also

$$\sum_{n=0}^{\infty}\frac{1}{n+1}\left(a_0^{\binom{n}{0}q^n}\, a_1^{\binom{n}{1}q^{n-1}}\ldots a_n^{\binom{n}{n}}\right)^{\frac{1}{(q+1)^n}} < \frac{q+1}{q}\log(q+1)\sum_{n=0}^{\infty}\frac{a_n}{n+1} \quad (2.1)$$

but here $\frac{q+1}{q}\log(q+1)$ is not the best possible constant [4]).

Both (2) and (2.1) reduce to identities for $q \to 0$.

Proof of (2):

$$\left.\begin{aligned}
&\sum_{n=0}^{\infty}\frac{1}{n+1}\left(a_0^{\binom{n}{0}q^n}\, a_1^{\binom{n}{1}q^{n-1}}\ldots a_n^{\binom{n}{n}}\right)^{\frac{1}{(q+1)^n}} \\
&\leqq \sum_{n=0}^{\infty}\frac{1}{n+1}\frac{1}{(q+1)^n}\sum_{m=0}^{n}\binom{n}{m}q^{n-m}a_m = \sum_{m=0}^{\infty}a_m q^{-m}\sum_{n=m}^{\infty}\frac{1}{n+1}\binom{n}{m}\left(\frac{q}{q+1}\right)^n \\
&= (q+1)\sum_{m=0}^{\infty}a_m\int_0^1\frac{t^m}{1+qt}\,dt < \sum_{m=0}^{\infty}a_m\int_0^1 t^m(q+1-qt)\,dt \\
&= \sum_{m=0}^{\infty}\frac{m+2+q}{(m+1)(m+2)}\,a_m.
\end{aligned}\right\} \quad (2.2)$$

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[2]) Loc. cit.

[3]) The best possible constant in this inequality lies thus between $\underset{0<x<1}{Max}\ \frac{1-x^2}{x}\log\frac{1}{1-x} = 1.082\ldots$ and $\pi-2 = 1.141\ldots$. The lower limit is obtained by taking $a_n = x^{2n}\ (0<x<1)$.

[4]) The best possible constant in (2.1) for $q=1$ lies between $\underset{0<x<1}{Max}\ x\frac{\log\frac{1}{1-x}}{\log\frac{1}{1-x^2}} = 1.260..$ and $2\log 2 = 1.386\ldots$, the lower limit being obtained by taking $a_n = x^{2n}$.

To prove that (2) is a sharp inequality it is sufficient to take $a_n = x^n$ $(0 < x < 1)$ and let $x \to 1$.

The proof of (2. 1) proceeds from (2. 2) as follows:

$$\sum_{n=0}^{\infty} \frac{1}{n+1} \left(a_0^{\binom{n}{0} q^n} a_1^{\binom{n}{1} q^{n-1}} \dots a_n^{\binom{n}{n}} \right)^{\frac{1}{(q+1)^n}} \leqq (q+1) \sum_{m=0}^{\infty} a_m \int_0^1 \frac{t^m}{1+qt}\, dt$$

$$= (q+1) \sum_{m=0}^{\infty} \frac{a_m}{m+1} \left(\frac{1}{q+1} + q \int_0^1 \frac{t^{m+1}}{(1+qt)^2}\, dt \right)$$

$$< (q+1) \left(\frac{1}{q+1} + q \int_0^1 \frac{t}{(1+qt)^2}\, dt \right) \sum_{m=0}^{\infty} \frac{a_m}{m+1} = \frac{q+1}{q} \log (q+1) \sum_{m=0}^{\infty} \frac{a_m}{m+1}.$$

Botany. — *On the influence of aggregation on the transport of asparagine and caffeine in the tentacles of Drosera capensis. L.* By W. H. ARISZ and J. OUDMAN. (Communicated by Prof. J. C. SCHOUTE).

(Communicated at the meeting of April 24, 1937).

§ 1. *Introduction.*

Most of the investigators who have occupied themselves with the carnivorous nutrition of Drosera are of the opinion that the tentacles play a part in the absorption by the leaf of the animal substances split by enzymes. (For literature see OUDMAN). MANGENOT also assumes a transport through the tentacles, and considers that aggregation has something to do with this. HOMÈS sees more connection between aggregation and secretion. More especially the phenomena of aggregation and secretion point to the substances being taken up by the tentacles. SCHMID and RUSCHMANN demonstrated that phosphorus and potassium were present in the tentacles after nutrition, whilst prior to it they were not so, or only in small quantities. A. KOK investigated the absorption of caffeine by the tentacles and the rapidity with which this substance penetrated from which she concluded that this process is a diffusion process. OUDMAN investigated the absorption of asparagine and caffeine quantitatively by means of the Micro-Kjeldahl method. His experiments showed that if agar with asparagine or caffeine were placed carefully on the marginal tentacles, the leaf a few hours later has become richer in nitrogen. In 24 hours with equimolecular solutions about 1½ times as much caffeine was taken up as asparagine. OUDMAN (l. c. p. 405) discusses the possibility of this being connected with the pathway of the movement of both substances, that for caffeine being more especially the vacuole and for

asparagine the protoplasm. The course of the absorption of asparagine is different from what would be expected in the case of a diffusion process. At 25° C. the nitrogen increase of the leaf presents a curious course when the heads of the tentacles are in a 1.5 % asparagine solution in agar 2 %. In table 1 we reproduce OUDMAN's tables 22 and 23 (l. c. p. 394 and 395). The absorption is seen to be weaker in the first period

TABLE I. Absorption of asparagine by marginal tentacles at 25° C. in the dark with leaves cut off. (OUDMAN 1936)

Treatment of the leaves	Time in hours	N increase in ‰ of the fresh weight			
					Without tentacles analysed
		I	II	III	IV
2 % agar + 1.5 % asparagine on the marginal tentacles	0 — 6	0.23	0.25	0.24	0.13
	6 — 12	0.70	0.64	0.76	0.83
	12 — 18	0.06	0.15	0.13	0.12
	18 — 24	0.13	0.11	0.11	0.03
	0 — 3	0.08	0.10		
	3 — 6	0.22	0.21		
	6 — 9	0.44	0.57		
	9 — 12	0.15	0.07		

of 6 hours than in the second period, whilst in the third and fourth periods practically no further absorption takes place. This is different from what one would expect in the case of a diffusion process; in that case the nitrogen increase ought to be greatest in the first period, and ought to decrease in the subsequent ones. In experiments in which the periods lasted 3 hours the same course of the absorption was also observed.

We considered it desirable to investigate this phenomenon more closely, and to make quantitative tests, for the sake of comparison, as to the course of the absorption of caffeine. We shall deal successively with (1) the course of the absorption of caffeine in the time, (2) the course of the absorption of asparagine in the time, as had already been investigated by OUDMAN, but with a method which seemed to us to be still more reliable, (3) the influence of aggregation on the absorption of caffeine and asparagine.

§ 2. *Method.*

All the tests were carried out with Drosera capensis. L. The method employed to determine the absorption by the tentacles agrees in the main with the one described by OUDMAN. The variability of the leaf material, which was caused partly by the varying ages of the leaves, was more or less eliminated in the same way, comparable series being obtained by working with a number of plants, one leaf of a different age being taken

from each plant for each series. From each of 4 plants 4 leaves were used, and from these 4 comparable series of leaf material were composed. One series served for the determination of the nitrogen content of the untreated leaves, the three others for the determination of the nitrogen content after various times. In this case, too, the amount of nitrogen determined was related to the fresh weight of the leaf. For the accuracy of this method cf. OUDMAN's table VI, p. 386.

The arrangement of the experiments was somewhat different. OUDMAN applied the agar with the tranport material by means of a syringe on to the extremities of the tentacles whilst it was still warm. The leaf surface rested on a slide. With this arrangement of the experiment there is a slight chance that transport substance, owing to capillary flow, may reach the leaf surface along the tentacles or the slide. In order to obviate this possibility the arrangement of the experiment was modified as follows. The agar with the transport substance was no longer squirted on to the tentacles in a liquid state. The warm agar, in which the transport substance was dissolved, was poured on to a glass plate, so that an agar plate was formed with a thickness of from 1.5 to 2 mm. From this strips were cut of approximately 7 mm. in width and about 5 cm. in length. Two of such strips were placed on a slide in such a way as to be somewhat further apart than the breadth of the leaf lamina. A leaf was laid between these, so that it rested with its marginal tentacles on the agar, whilst the leaf surface was in the air. The actual leaf lamina is in this way in contact neither with the slide nor with the agar. A strip of glass was then placed longitudinally over the middle of the leaf, and was pressed firmly into two pieces of plasticine attached to both ends of the slide. Finally two strips of agar, of the same shape and composition as above, were laid at the extremities of the marginal tentacles so that these were between the strips of agar, and were enabled to absorb the substance dissolved in it (cf. fig. 1).

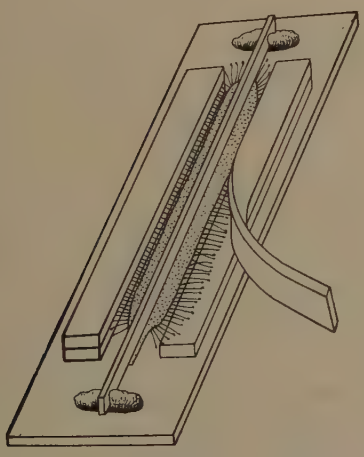

Fig. 1. Leaf of Drosera capensis with the marginal tentacles between strips of agar in which asparagine or caffeine has been dissolved.

This arrangement of the experiment has several advantages. In the first place the agar is in this way applied cold to the tentacles, so that too high a temperature is not to be feared. When agar was squirted on there was always a danger of its coming into contact with the leaf lamina, which now cannot occur. In the third place the leaf lamina with this arrangement is not in contact with the agar on

the slide, thus preventing any possible spreading of the transport substance out of the agar over the surface of the slide to the leaf. The amount of substance applied is further in all cases the same. And finally there is this advantage also, that more tentacles have an opportunity to take something up. From 120 to 150 tentacles now participate in the absorption, as against 80—90 with the former method.

§ 3. *Absorption of caffeine by the tentacles.*

As already stated in the introduction, experiments as to the transport of caffeine in the tentacles of Drosera capensis were carried out by A. KOK. Her experiments show that the rapidity of transport is directly proportional to the square root of the time, from which she draws the conclusion that we have here an ordinary diffusion phenomenon in the vacuole, which, however, is retarded by the passage of the protoplasm from cell to cell.

The question was now how absorption and transport of this substance proceed with a quantitative investigation. The marginal tentacles were

TABLE II. Absorption of caffeine by the marginal tentacles at 25° C. in the dark

Treatment of the leaves	Time in hours	N increase in ‰ of the fresh weight							Average
		I	II	III	IV	V	VI	VII	
Marginal tentacles in 2% agar + 1% caffeine	0 — 4	0.51	0.34	0.23	0.60	0.75	0.56	0.52	0.50
	4 — 8	0.23	0.30	0.16	0.19	0.37	0.09	0.35	0.24
	8 — 12	0.07	0.07	0.29	0.14	0.20	0.15	0.17	0.16

placed between strips of agar 2 %, in which 1 % caffeine was dissolved. Table 2 shows that the course of the absorption is in entire agreement with that of a diffusion process. During the first four hours more caffeine is taken up than in the following four hours, and during the last period of four hours the smallest amount is taken up. Possibly owing to the fact that the material used was not completely regular, the results are somewhat divergent. But the average values of the whole series show a satisfactory course. With a quantitative investigation also, therefore, we come to the conclusion that caffeine in the Drosera tentacles is transported in accordance with a process which may be compared to a physical diffusion.

§ 4. *Absorption of asparagine by the tentacles.*

OUDMAN's former determinations as to the absorption of asparagine are found in table 1. The new determinations with the modified method are resumed in table 3. The absorption was determined in these tests in periods of four hours. The concentration of the asparagine also amounted to 1½ %. In general the absorption in these experiments is seen not to

TABLE III. Absorption of asparagine by the marginal tentacles at 25° C. in the dark.

Treatment of the leaves	Time in hours	N increase in ‰ of the fresh weight				
		I	II	III	IV	V
Marginal tentacles in 2 % agar + 1½ % asparagine	0 — 4	0.08	0.14	0.25	0.08	0.25
	4 — 8	0.31	0.43	0.35	0.13	0.52
	8 — 12	0.12	0.18			

be particularly marked. The course of the absorption shows the same picture as previously. A weak absorption in the first period, followed by a stronger absorption in the second period, whilst in the third period still less was taken up.

§ 5. *Transport by the tentacles to the leaf.*

The question now arises whether the caffeine and asparagine absorbed are also transported to the leaf. Caffeine will to a great extent be present in the vacuole of the tentacle cells owing to its being transported by a diffusion process. The question is to what extent caffeine also finds its way into the leaf itself, since it is about four hours before the caffeine can be diffused through the long marginal tentacles in the leaf itself. For this reason an experiment was made in which, at the end of the transport, the marginal tentacles were cut off, so as to permit of determining whether absorption of nitrogen had also occurred in the leaf itself, to which the stumps of the cut-off tentacles were still attached. A test of this kind was carried out with 1½ % asparagine. Table 4 contains the results of

TABLE IV. Absorption of asparagine and caffeine by the marginal tentacles. The marginal tentacles cut off before the analysis.

Treatment of the leaves	Time in hours	N increase in ‰ of the fresh weight		
		I	II	III
Marginal tentacles in 2 % agar + 1 % caffeine	0 — 4	0.20	0.19	0.14
	4 — 8	0.04	0.00	0.00
Marginal tentacles in 2 % agar + 1½ % asparagine	0 — 4	0.35	0.12	0.08
	4 — 8	0.30	0.40	0.27

these tests. It is seen that the leaf with the tentacle stumps has actually absorbed some caffeine in the first four hours. In the second period of four hours the increase is so slight as not to be demonstrable.

With the asparagine test, on the other hand, very little asparagine was taken up in the first four hours (except with the first test, which gave a divergent result), whilst in the second period of four hours a considerable increase took place. Whilst therefore the transport of caffeine to the leaf

is slight, it is seen that more asparagine has got into the leaf in eight hours. This strengthens the impression that asparagine is transported in a different way from caffeine. This difference may be due to asparagine not permeating into the vacuole. Experiments to determine the permeability of asparagine into the vacuole of the tentacle cells are difficult to carry out. From comparative plasmolysis tests of cells in the base of the tentacles with 8 % glucose and 8 % glucose plus 1½ % asparagine we consider ourselves justified in concluding that asparagine does not penetrate into the vacuole, or only very slowly. This would seem to indicate that for the asparagine transport only parts of the cell outside the vacuole are available. It is an obvious supposition that the transport occurs more especially in the cytoplasm. It is remarkable that notwithstanding the fact that the pathway of transport is so small more asparagine than caffeine is transported through the tentacles.

With regard to the possibility that the transport of asparagine and caffeine takes place outside the living cells, through the centrally situated spiral vessel we made an odd experiment. For this one of two series of three leaves each was placed in moist air in the usual manner, whilst the petioles of the leaves of the other series were in water, so that suction forces which might cause a suction through the spiral vessels of the tentacles were eliminated. The experiment lasted eight hours; with both series the same amount of asparagine, viz. 0.43 $^0/_{00}$ and 0.45 $^0/_{00}$ nitrogen of the fresh weight was taken up.

§ 6. *Influence of aggregation on the transport of caffeine and asparagine.*

It is known that a large number of substances are able to cause the phenomenon of aggregation in the tentacle cells. By aggregation is understood a process with which the following changes occur in the tentacle cells. In the first place increased protoplasmic streaming, in the second place increase of the volume of the cytoplasm and decrease of the volume of the vacuole, in the third place partition of the vacuole. These changes may occur separately in the cell without anything that can be called aggregation. Increased streaming of protoplasm occurs with chemodinesis. With vacuole contraction there is an increase of the volume of the cytoplasm. This phenomenon is repeatedly to be observed in the case of tentacle cells (KEIL). We cannot here enter into the connection between these processes and aggregation. Substances which cause aggregation may according to DARWIN, SCHMID, ÅKERMAN and COELINGH be anorganic salts, such as e.g. KH_2PO_4, organic nitrogenous substances such as pepsin, asparagine etc., and organic substances free from nitrogen such as e.g. salicin. In contrast with asparagine caffeine does not cause aggregation; this substance even prevents the occurrence of aggregation or causes an aggregation that has already occurred to disappear (ÅKERMAN).

a. Transport of asparagine in aggregated tentacles.

It seemed desirable to use substances for the aggregation which did not contain nitrogen. The choice fell on salicin ¼ % and KH_2PO_4 $^1/_{10}$ %. The aggregation took place by putting the tentacles of the leaves in the same way as with the transport test between strips of agar in which one of the two substances causing aggregation is present. Here the difficulty presents itself that the tentacles begin to bend in a marked degree as soon as they are removed from the agar, so that it is difficult to bring them again under the agar with the transport substance in the next part of the experiment. With some experiments an attempt was therefore made to obviate this difficulty by means of cooling in a refrigerator. Against this, however, there was the objection that the transport often was less strong, probably as a result of the preceding cooling-off. By working quickly we managed, however, to get the aggregated tentacles between the agar plates with asparagine. With the first tests aggregation was continued for 12 hours with ¼ % salicine, after which agar + asparagine 1½ % was applied and the increase of nitrogen was determined after four, eight, and twelve hours. Table 5 gives an impression of the course of

TABLE V. Influence of aggregation on absorption of asparagine by the marginal tentacles at 25° C. in the dark.

Treatment of the leaves	Time in hours	N increase in ‰ of the fresh weight			
Agregation		Salicin 1/4 %			KH_2PO_4 0.1 % at low temperature
		11 hours	12 hours	12 hours	16 hours
Marginal tentacles in 2 % agar + 1½ % asparagine	0 — 4	0.36	0.52	0.46	0.36
	4 — 8	0.42	0.22	0.27	0.15
	8 — 12	0.22	0.10		

the increase of nitrogen. It is remarkable that with aggregated tentacles in all the experiments as much or more was taken up in the first period of four hours than in the second period of four hours. A change must have occurred in the tentacles owing to the aggregation, in consequence of which the absorption is immediately strong and gradually decreases in the following periods. An experiment with different duration of aggregation, the transport after four and sixteen hours aggregation being compared, showed no important differences.

Since asparagine itself is also able to cause aggregation, it is probable that the stronger absorption of asparagine in the case of the non-aggregated tentacles in the second period is caused by the fact that an aggregation is brought about during the first period, and the tentacles therefore behave in the second period like aggregated tentacles.

b. Transport of caffeine in aggregated tentacles.

The rapidity of the caffeine diffusion in aggregated tentacles has already been determined by A. KOK. She found in 24 hours with pepsin aggregated tentacles a feeble inhibition of the caffeine transport. We also repeated these experiments with other substances causing aggregation. Without going into details as to this here, we can confirm her results. We found, however, that according as the aggregation lasts longer the differences between aggregated and non-aggregated tentacles are smaller.

The quantitative investigation of the influence of aggregation on the caffeine-transport was also carried out with salicin and KH_2PO_4 as aggregating substances. Owing to the curving of the very active tentacles these experiments can lay no claim to any great degree of accuracy; they give the impression that aggregation has no influence on the caffeine transport.

§ 7. *Discussion of the results.*

We were able to confirm OUDMAN's observations that both caffeine and asparagine is taken up and transported by the tentacles. As previously found by A. KOK caffeine is taken up in the same way by normal tentacles as by aggregated ones. The process resembles an ordinary diffusion process through the vacuole. A considerable part of the caffeine remains in the tentacles, whilst only a part of it is found in the leaf itself.

The absorption of asparagine proceeds differently in the case of aggregated tentacles than with non-aggregated ones. With the former the absorption is considerable in the first period, and in the next period is equally large or smaller, afterwards decreasing. With the non-aggregated tentacles the absorption is slight in the first period and afterwards stronger. A previous aggregation therefore results in a better absorption and stronger transport. As to the manner in which the aggregation causes this we are quite in the dark. Various possibilities suggest themselves. One possibility is that in the case of the aggregated tentacles the permeability of the tonoplast for asparagine is increased. As, however, with the caffeine transport it is rather an inhibition which occurs, this conflicts with the hypothesis of an increase in the permeability of the tonoplast. The asparagine is therefore probably transported through the cytoplasm in the case of the aggregated tentacles also. The acceleration of the transport by the aggregation might be due to one of the phenomena which occur with the aggregation, whether to the increase in the size of the pathway of transport owing to the swelling of the protoplasm or to the accelerated streaming of protoplasm or perhaps to both causes. We will content ourselves with indicating these possibilities here; only by further experiment will it be possible to elucidate this point.

We wish heartily to thank the Natural Science Section of the Kon.

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Summary.

The absorption of caffeine by the tentacles of Drosera capensis displays a similarity to an ordinary diffusion process. The vacuole serves as the principal pathway of transport. The absorption of asparagine depends on the condition of the tentacles; with normal tentacles little is taken up in the first period of from four to six hours, but more in the succeeding hours; with aggregated tentacles a good deal is immediately absorbed. The principal pathway of transport for asparagine is most probably the cytoplasm. An ordinary physical diffusion cannot account for this transport owing to the comparatively slight amount of protoplasm.

Groningen, April 1937. *Laboratory for Plant Physiology.*

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Botany. — *On the transport of introduced nitrogenous substances in the leaves of Vallisneria spiralis.* By W. H. ARISZ and J. OUDMAN [1]). (Communicated by Prof. J. C. SCHOUTE).

(Communicated at the meeting of April 24, 1937).

§ 1. *Introduction.*

The object of this investigation is the quantitative determination of the transport in leaves of Vallisneria spiralis. The BIERBERG experiments with regard to the influence of the protoplasmic streaming on the transport in the leaves of this plant were repeated at this laboratory by A. KOK. She found that caffeine and lithium salts taken up by the leaf from outside show a retarded diffusion. No acceleration caused by the protoplasmic streaming was demonstrable. The transport substances employed were substances which do not naturally occur in Vallisneria leaves, whilst the method of analysis used did not admit of a quantitative determination. In view of our experiments with Drosera tentacles, in which an accelerated transport for asparagine with a different course from the transport of caffeine was found, it seemed advisable to investigate quantitatively the transport of asparagine and caffeine in the case of Vallisneria leaves also. Asparagine is so suitable for transport tests owing to its being a substance which in natural conditions may occur in a plant.

§ 2. *Method of Investigation.*

The method employed in these investigations is based entirely on that of A. KOK with Vallisneria and on that of our Drosera tests. In the experiments of A. KOK, pieces of Vallisneria leaves took up from agar, in which the transport substance was to be found, the dissolved substance to a sufficient extent at one extremity (KOK, fig. 3, p. 54). In our experiments the transport substances used were asparagine and caffeine, both nitrogenous substances, which were applied in a 2 % agar. The concentration of both substances in all the experiments was 0.05 mol. It was possible to demonstrate absorption and transport by the increase in the nitrogen content of the leaves. All nitrogen determinations were carried out by the micro-Kjeldahl method with the Parnas-Wagner apparatus. An increase of nitrogen is only to be determined with sufficient accuracy if the nitrogen content of the leaf is known. Orientation tests showed the nitrogen content of the leaf not to be equal in all parts. The

[1]) Dr. OUDMAN wishes to acknowledge his indebtedness for a grant of the „Stichting tot verruiming van werkgelegenheid voor academisch gevormden".

content is greatest at the top, and gradually decreases towards the base, the greatest differences being more than 50 %. In order to eliminate these differences in one and the same leaf and those between various leaves, the following plan was adopted. (Cf. fig. 1). Leaves of suitable length

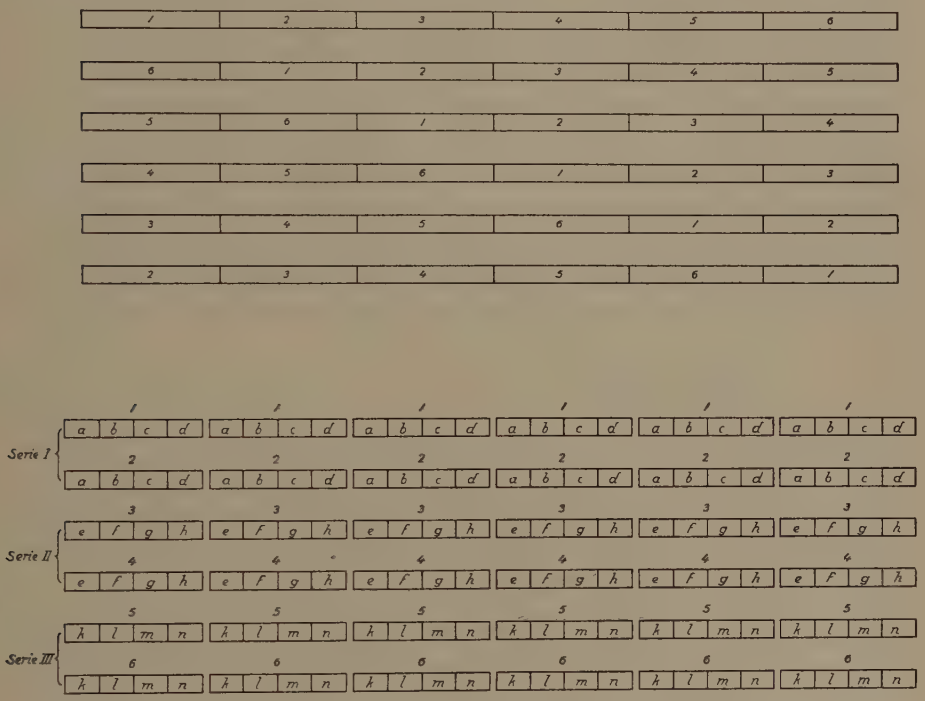

Fig. 1. Method of combination of pieces of leaves in order to eliminate the variability between the leaves and between different parts of a single leaf.

were cut off, divided into 6 equal parts of 3,5 cm. and numbered as shown in fig. 1. 6 Leaf pieces no. 1 together give a combination from all parts of the leaf, top, middle, and base, but at the same time composed of 6 different leaves. One series is formed by two such combinations of 6 leaf pieces. Series consisting of 12 of such leaf pieces still have too great a variability, which is caused by the leaves differing in breadth from one another and by their further being broader at the top than at the base. For this reason the leaves were cut off along the sides to a uniform breadth of 4 mm.

It was necessary for the transport experiments to apply the transport substance at one extremity of the leaf piece and to determine how the nitrogen of the rest of the leaf increased. For this purpose the 3.5 cm. long pieces were transversely divided into small pieces of 8 mm. by means of a brass plate in which grooves had been made, through which a razor-blade could be moved. Narrow strips were cut away at both extremities, so that 4 small pieces of 8 mm. came from one leaf piece (see fig. 1),

the small pieces *a*, *b*, *c*, *d* from series I, the small pieces *e*, *f*, *g*, *h* from series II, and the small pieces *k*, *l*, *m*, *n* from series III. In some experiments longer leafpieces were investigated; these, also, were divided into small pieces of 8 mm. The first series of 12 leaf pieces thus yields 12 small pieces *a*, 12 small pieces *b*, etc.; the second series 12 small pieces of *e*, *f*, etc., the third series 12 small pieces of *k*, *l*, etc. In each case 12 small pieces together were analysed to find their content of nitrogen. Series I served as check, series II and III served for transport tests; the strength of absorption and transport was found by determining the difference in nitrogen of the pieces of series II and III with the average amount of nitrogen in the pieces of series I. The application of the transport substances asparagine or caffeine took place in agar 2 %. Plates of about 3 mm. thickness were cast of this, and these were cut into strips of 8 mm. broad. With the experiments carried out later, the ends of the leaf pieces projected somewhat out of the agar, as they were found in that case better to remain turgescent (cf. p. 444). Both on the lower and on the upper side a strip of agar of this kind was applied over the first zone of 8 mm. of the leaf (see fig. 2). The other extremity of the leaf was in

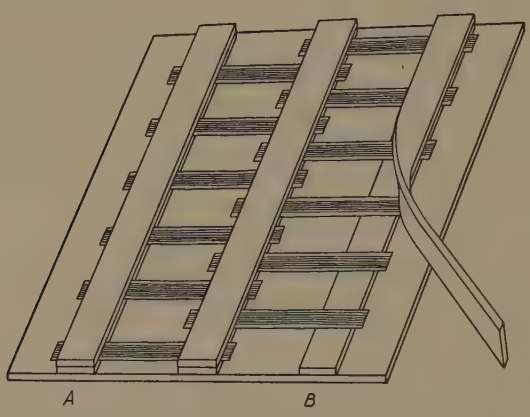

Fig. 2. Pieces of leaves of Vallisneria spiralis laid between agar strips for the diffusion tests. The agar strips A and B contain the transport substance.

contact with a strip of pure agar on the upper and lower side. On the conclusion of the transport experiment the leaf pieces were washed and divided as indicated above into pieces 8 mm. in length.

The reliability of the method described is satisfactory. The differences in the amount of nitrogen of the pieces *a*, *b*, *c*, *d* is at the most from 3 to 4 %. With experiment 32, for instance the amount is in γ nitrogen 144, 148, 142, 146, average 145 γ, with experiment 33 in γ nitrogen 138, 138, 134, 138, av. 137 γ, and with experiment 35 144, 138, 138, 142, av. 140.5 γ. With experiment 34 longer leaf pieces were divided into 7 small pieces of 8 mm. The checks not treated (8 small pieces) contained 110, 110, 108, 110, 112, 112 and 114 γ nitrogen. A difference greater than

6 γ nitrogen therefore indicates a transport in the leaf. As one series consists of 12 small pieces, the demonstrable increase per small piece is only $\frac{1}{2}$ γ.

The experiments were carried out from the month of November 1936 to March 1937. The plants grew much more vigorously in February and March.

§ 3. Transport tests with asparagine and caffeine.

A commencement was made with leaf pieces of 35 mm., from which 4 pieces of 8 mm. were cut. The experiments were carried out in a room in the laboratory in daylight. The leaves were cut into pieces, after which they were allowed to remain from 15 to 20 minutes in a Petri dish on filter paper and arranged for the transport tests in the manner indicated, in glass boxes with a diameter of 12 cm. and a cubic capacity of 500 cc., in which the air was saturated with water vapour. The boxes were placed during the transport in a room with a constant temperature of 25° C. in the dark.

The results of the asparagine tests are summarized in table 1. With each test the number is given. Tests with the same number are parallel tests, which are distinguished by the addition of A. B. C. etc. The duration of the test is shown, and usually amounts to from 18 to 24 hours. The increase of nitrogen is expressed in γ, and is calculated on a leaf surface of $12 \times 8 \times 4$ mm^2. $=384$ mm^2. In some cases (tests 27, 34 and 42) 8 leaf pieces were analysed. These values have been converted to the same area as in the other experiments.

From table I it is seen that the total absorption of nitrogen in the leaf

TABLE I. Transport of asparagine 0.05 mol. at 25° C. in the dark. Amount of nitrogen taken up by pieces of leaves in γ. Pre-treatment 15—20 minutes in water with filter paper.

Number of experiment	15 A	21 A	24	26	30	34
Time of transport in hours	22 h.	22 h.	23 h.	24 h.	22 h.	22 h.
First piece of 8 mm.	85	96	54	82	86	73
Second piece of 8 mm.	41	48	34	36	52	52
Third piece of 8 mm.	31	40	18	16	20	25
Fourth piece of 8 mm.	39	48	34	26	30	19
Total amount	196	232	140	160	188	169

pieces was considerable, 140—232 γ nitrogen, i.e. 658—1090 γ asparagine. The distribution of the nitrogen over the leaf is not quite according to expectations. One would expect most would be found in the first piece which is in direct contact with the asparagine, and that the amount of nitrogen would gradually decrease towards the other end. It is true that the nitrogen increase is in all cases greatest in the first piece,

but the decrease is not gradual, since the last piece, except in experiment 34, shows a greater nitrogen increase than the last but one. Experiment 34 was carried out with longer leaf pieces of 60 mm., from which 7 small pieces of 8 mm. were cut; the last 3 small pieces showed no increase of nitrogen. This indicates that the accumulation in the other experiments in the last piece is a marginal effect. By means of a modification in the method we succeeded in eliminating this marginal effect at any rate partially. In this first series of experiments the last piece of the leaf had its edge entirely surrounded by the agar, so that the intercellular tubes, which had been opened by the cutting, were closed. Leaves of this kind showed some tendency to infiltration of the intercellulars and slighter turgescence of the parenchym. The phenomenon was especially strong when both ends were altogether in agar. In the later experiments (experiment 36 et seq.) we therefore always left the edge of the leaf pieces projecting somewhat from the agar. (cf. fig. 2). The leaves then retained their turgescence better, and the marginal effect was less noticeable. In so far as this marginal effect was due to injury it was impossible to eliminate it, unless the experiment was postponed until the effect of the injury had disappeared.

The transport experiments with caffeine had a somewhat different course. Table II gives a summary of these tests. Tests 15 A and B are

TABLE II. Transport of caffeine 0.05 mol. at 25° C. in the dark. Amount of nitrogen taken up in γ.

Pre-treatment	15—20 min. water-filter paper		$2^1/_2$ h.	4 h.
Number of experiment	15 B	21 B	20 B	23 B
Time of transport in hours	22 h.	22 h.	18 h.	$19^1/_2$ h.
First piece of 8 mm.	112	138	84	175
Second piece of 8 mm.	49	52	34	71
Third piece of 8 mm.	9	8	—	—
Fourth piece of 8 mm.	—	18	—	—
Total amount	170	216	118	246

parallel tests. It is a striking fact that caffeine is well absorbed, but less strongly transported; experiments 21 A and B also give a similar result, but here the diffusion of caffeine was somewhat stronger, so that here, too, the marginal effect occurs.

The result of the above tests is therefore that caffeine and asparagine are both absorbed by the Vallisneria leaf from agar, but that the transport of asparagine is somewhat different from that of caffeine, being less pronounced in the case of caffeine.

In this publication we will further occupy ourselves exclusively with the transport of asparagine. The influence of the pretreatment of the

leaf pieces on the subsequent transport was investigated, whilst a few orientation tests were made on the effect of temperature at which transport takes place. It was further determined to what extent the transport is polar. In all the experiments, with the exception of test 27, the direction of transport was from the top to the base of the leaf.

§ 4. Influence of the pretreatment on the transport of asparagine.

The tests summarized in table I agree well with one another. As it happened, however, the time that the leaf pieces remained in trade distilled water before the setting up of the experiment was considerably lengthened in the case of one experiment. In this case a much slighter transport was found. It was this which led us to make a more detailed investigation of the influence of the previous treatment. Table III contains

TABLE III. Transport of asparagine 0.05 mol. at 25° C. in the dark. Amount of nitrogen taken up in γ.

Pre-treatment hours in	Dist. water with filter paper						
	2 h.	2 h.	2½ h.	3 h.	4 h.	4 h.	4 h.
Number of experiment	33 A	40 A	20 A	31	23 A	32	33 B
Time of transport in hours	21 h.	21½ h.	18 h.	19 h.	19½ h.	19 h.	21 h.
First piece of 8 mm.	49	68	38	136	37	155	87
Second piece of 8 mm.	25	38	10	74	15	39	39
Third piece of 8 mm.	—	20	—	74	—	11	25
Fourth piece of 8 mm.	—	12	—	84	—	7	19
Total amount	74	138	48	368	52	212	170

the results of tests in which the leaf pieces remained for from 2 to 4 hours in trade distilled water on filter paper before being used for the transport tests. The results of these tests show a deviation from those of table I. With a short pretreatment the transport is less strong. With about 4 hours in water the transport in the case of test 23 was slight, with the others, however, it was strong, even stronger than with the tests described in table I.

In order to enable a closer study of this phenomenon to be made, the leaf pieces were put during the pretreatment into tap water distilled over glass, and came into contact with filter paper only when being dried. The duration of this pretreatment was from 20 minutes to 19 hours. Table IV contains the results of these tests. The results are much more regular than those of table III; the longer the pretreatment lasts, the stronger is the transport. The marginal effect mentioned in the previous § has here practically disappeared, owing to the modification of method, except in the case of the tests with short treatment in water, tests 41 and 39 A and B. Test 44 (table IV) was carried out with somewhat longer leaf pieces, from which 5 small pieces were cut.

Test 42 A shows a very strong absorption and transport in table V. In this case leaf pieces of 60 mm. were divided into 7 pieces of 8 mm.

TABLE IV. Transport of asparagine 0.05 mol. at 25° C. in the dark. Amount of nitrogen taken up in γ.

Pre-treatment hours in pure water	20 min.	1 h.	2 h.	2 h.	4 h.	4 h.	12 h.	19 h.
Number of experiment	41 C	39 A	39 B	40 B	39 C	41 A	41 B	44 A
Time of transport in hours	24 h.	21 h.	21 h.	21½ h.	21 h.	24 h.	24 h.	24 h.
First piece of 8 mm.	61	90	118	92	136	99	133	107
Second piece of 8 mm.	19	38	74	56	66	61	83	71
Third piece of 8 mm.	19	16	24	24	42	39	39	37
Fourth piece of 8 mm.	23	20	34	20	34	29	33	29
Fifth piece of 8 mm.								19
Total amount	122	164	250	192	278	228	288	263

Altogether 556 γ nitrogen was absorbed. A remarkable feature is the very gradual decrease in the transport from the point of absorption.

In connection with FITTING's investigations of chemodinesis in Vallisneria the influence of a pretreatment of 45 minutes and of one of 4 hours in a 0,00001 % asparagine solution was determined. With the first test a total of 170 γ nitrogen was absorbed, distributed over 4 leaf pieces 90, 34, 14, 32; with the second test 184 γ nitrogen was taken up, distributed as follows: 87, 35, 27, 35. With both tests in the leaf pieces pre-treated with asparagine 0.00001 % the same amount or somewhat less was absorbed than in the checks, which were pretreated with distilled tap-water.

§ 5. Influence of temperature on transport.

Only a few orientation tests can here be reported; they are summarized

TABLE V. Influence of temperature on transport of asparagine 0.05 mol. Amount of nitrogen taken up in γ.

Temperature of transport	5° C.	5° C.	25° C.	35° C.
Number of experiment	44 B	43	42 A	42 B
Pre-treatment hours in pure water	19 h.	18 h.	18 h.	18 h.
First piece of 8 mm.	49	37	178	196
Second piece of 8 mm.	19	—	106	127
Third piece of 8 mm.	13	5	70	106
Fourth piece of 8 mm.	—	—	58	100
Fifth piece of 8 mm.	—	8	49	77
Sixth piece of 8 mm.			49	64
Seventh piece of 8 mm.			46	55
Total amount	81	50	556	725

in table V. At 5° C. there is a distinct absorption in the first leaf piece, but the transport is slight; at 35° C. the transport is still greater than at 25° C. in the parallel test just discussed. The total amount here accumulated by a leaf surface of 2688 mm². in 18 hours is 725 γ nitrogen i.e. 3.4 mgr. asparagine. Of this $^3/_4$ was transported to other parts of the leaf, whilst the absorption took place through $^1/_7$ of the total leaf area, that is 384 mm².

§ 6. Influence of polarity on transport.

From test 27 described below it is seen that the transport is not polar. Leaf pieces 60 mm. long were placed with the middle zone between two agar strips 8 mm. broad, containing asparagine. Transport of the asparagine taken up in the middle of the leaf piece took place towards both ends. The pre-treatment of this test was from 15 to 20 minutes in trade distilled water with filter paper. The transport lasted 22 hours. After this time there were 84 γ nitrogen in the middle zone of the leaf, in the zones towards one end 60, 51, and 54 γ, and in the one towards the other extremity 69, 42, and 63 γ.

§ 7. Discussion of the results.

It has been shown in the above that a transport may take place in the cut-off leaf pieces of a Vallisneria leaf, and that this can be determined quantitatively with the aid of the Micro-Kjeldahl analysis method. The transport is not polar and dependent on the temperature during the transport and on the previous treatment. The question as to whether the asparagine absorbed is transported unchanged in the leaf, or whether it is transformed, is left out of discussion here.

With the preliminary experiments it was found that asparagine is transported better than caffeine, and that the accumulation in the leaf from the point of absorption decreases much more gradually with the asparagine experiments than with the caffeine experiments. Consequently asparagine is better transported than caffeine.

With longer leaf pieces a stronger absorption was found than with shorter ones. This points to the absorption being increased by stronger transport.

The principal question to be discussed here is which channels the transport takes place through. With the experiments of A. KOK the possibility was discussed that substances might be transported through the numerous air-channels of the leaf. She was able to show by experiments that no liquid can be pressed through the air-channels even under pressure. (KOK p. 70). SOLEREDER states that diaphragms consisting of one cell-layer occur in the channels with small intercellulars. It is extremely improbable that transport is possible through these partitions. Nor are the results of our experiments in agreement with the hypothesis of a transport through the air-channels. The transport is best precisely when

the air-channels are filled with air. The strong influence of the temperature, the difference in behaviour of caffeine and asparagine point to the cooperation of living cells. We also consider a transport along surface layers outside the living cells to be out of the question.

To what extent the transport occurs through particular tissue-elements cannot, however, be elucidated on the strength of the above experiments. The bundles of the Vallisneria leaves are but slightly differentiated. There are no water conducting vessels whatever.

The question as to whether there is any connection between protoplasmic streaming and the transport found cannot be discussed here either. It is remarkable in this connection that the pre-treatment has a great influence, but a previous treatment with substances which cause a rotation of protoplasm, such as filter paper, trade distilled water which may contain copper, and asparagine yields no appreciable result. The longer the leaves have remained in pure water during the pre-treatment, the more strongly they transport. This will partly depend on the recovery of the leaf after the injury. FITTING also found that the streaming of protoplasm excited in the cut-off leaf pieces gradually comes to a standstill, but it is certain that the state of turgescence also has something to do with it. After a long pre-treatment in pure water the leaf pieces were found to remain turgescent better during the transport tests than after a short pre-treatment. The infiltration of the air-channels too was slighter in that case. It is remarkable that BOTH also found in his transport tests in simple systems in the case of Impatiens Marianae, that the transporting leaf has to be turgescent in order to be able to transport substances to the bundles.

With transport tests with the tentacles of Drosera we also found a difference in behaviour between asparagine and caffeine. Asparagine is there probably transported by the protoplasm, whilst caffeine penetrates into the vacuole. The data hitherto obtained with regard to the transport in the case of Vallisneria may perhaps indicate that the transport of asparagine with these plants also is an accelerated transport in protoplasm.

Summary.

With the aid of the Micro-Kjeldahl analysis method the transport of asparagine and caffeine was investigated in the case of Vallisneria spiralis. A method was worked out to make the individual differences of the leaf pieces in nitrogen content so small that transport of nitrogenous substances containing more than 6 γ nitrogen ($\frac{1}{2}$ γ per small piece) could be demonstrated. In successive pieces of 8 mm. in length the nitrogen uptake decreases gradually from the point of absorption; for asparagine the transport is stronger than for caffeine.

It was possible to eliminate a marginal effect.

The following factors were found to have an influence on the transport of asparagine. 1°. the temperature during the transport, 2°. the pretreatment of the leaf pieces before the transport begins; the injury by cutting the leaf, impure water and the length of time for which the leaf pieces remain in pure water are of influence, 3°. the length of the leaf pieces in which the transport takes place.

It is probable that the transport found takes place in living cells and is comparable to the transport demonstrated by us in the tentacles of Drosera.

Groningen, April 1937. *Laboratory for Plant Physiology.*

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Palaeontology. — *Sur une espèce nouvelle du genre Sabinia (Caprininés).* Par L. A. H. BOUWMAN. (Communicated by Prof. L. RUTTEN.)

(Communicated at the meeting of April 24, 1937).

Au bout de l'année 1935 M. H. KUGLER, géologue à Trinidad, envoya à l'Institut Géologique de l'Université d'Utrecht une collection de fossiles, provenant de Point à Pierre (Trinidad).

Ces fossiles ont été recueillis d'un bloc isolé d'un conglomérat calcaire. La collection se compose de Rudistides pour la plupart très fragmentaires et, en outre, de quelques fragments de coraux, de *Nerinea* et d'une (?) *Ostrea*, pas encore indentifiés.

Presque tous les Rudistides appartiennent à une nouvelle espèce du genre *Sabinia*. Le reste, treize fragments d'un Pachyodonte, qui se rapproche beaucoup de *Matheronia*, paraissent appartenir à un nouveau genre, qui sera décrit plus tard.

Entre quelques centaines de fragments, nous n'avons trouvé qu'une vingtaine de fragments de valves inférieures (dont une assez complète), tandisque la valve supérieure est représentée par environ 100 fragments, dont une vingtaine assez complètes. D'une centaine de fragments il est impossible de dire à laquelle des deux valves ils appartiennent.

Presque l'entière collection se trouve maintenant dans le musée d'Histoire Naturelle de Bâle; huit valves supérieures (pour la plupart très fragmentaires) et deux fragments de valves inférieures ont été retenus à Utrecht.

Description de Sabinia kugleri (n. sp.)

Rudistides fixées, du type inverse. Valve inférieure plus grande que la valve supérieure. Valve supérieure très courbée, valve inférieure par contre à courbure faible.

La couche extérieure a disparu chez tous les individus; la couche intérieure consiste en canaux polygonaux, avec une rangée marginale de canaux pyriformes, qui n'est conservée que dans un petit nombre d'exemplaires.

Valve inférieure. La valve inférieure irrégulièrement conique, quelquefois un peu tordée, est courbée un peu vers le côté dorsal (fig. 3). La rainure ligamentaire est peu frappante, mais pourtant bien visible.

La cavité d'habitation profonde est relativement petite; sa section est elliptique, parfois à peu près circulaire.

Cavité AII′ en profil elliptique ou trigone à coins arrondis. Un individu présente dans AII′ des cloisons très distinctes. Canaux polygonaux cloisonnés.

La cavité PIV′ à section très typique, quoique variable (voir fig. 5a, b, et c), n'est pas remplie tout à fait par la dent PIV.

La dent 3 s'est cassée plus ou moins chez tous les exemplaires, de sorte qu'on ne peut dire rien de certain sur sa forme.

Les lames myophores ne sont plus visibles; probablement elles étaient saillantes et à cause de cela cassées (pour leur forme probable cf. *Praecaprina varians* et *Offneria rhodanica*, dans: PAQUIER, Les Rudistes Urgoniens, 1905, Mém. Soc. Géol. de Fr., 2me partie, pl. XVII).

Valve supérieure. La forme extérieure est toujours capuliforme. Comme chez la valve inférieure, la couche extérieure a disparu. La partie extérieure de la couche intérieure montre parfois des lignes d'accroissement.

L'impression myophore postérieure est développée en lame saillante, formant en quelque sorte un prolongement postérieur, élevé et étroit de la dent PIV jusqu'au delà de la cloison entre la cavité 3′ et la cavité d'habitation (fig. 1).

Entre les dents PIV et AII se trouve la cavité 3′, isolée de la cavité d'habitation par une cloison courte.

La cavité 3′, quoiqu'elle ne puisse pas avoir été remplie par la dent 3 tout à fait, ne s'étend pas dans une cavité accessoire nette. En conséquence la cloison diviseuse est courte, comparée à celle des autres espèces.

Dans ladite cavité 3′ on peut distinguer deux parties différentes: une partie profonde à côté de la cavité d'habitation et un autre peu profonde avec plusieures embouchures de canaux, qui devient peu à peu moins profonde vers le côté dorsal (cf. *Schiosia ramosa*, dans: DOUVILLÉ, Sur quelques Rudistes Américains, Bull. Soc. Géol. de Fr., 3me série, Tome XXVIII, pag. 208).

A côté de la dent AII se trouve l'impression myophore antérieure consistant en un plateau un peu élevé, percé de canaux et couvert de nodules arrangées radialement, qui forment en quelque sorte les séparations entre les canaux eux-mêmes.

Comparaison avec les autres espèces du genre Sabinia.

Dans la littérature les espèces suivantes sont mentionnées:

Sabinia sublacensis	PARONA	(1908)
„ *aniensis*	PARONA	(1908)
„ *sinuata*	PARONA	(1908)
„ *klinghardti*	JOHS. BÖHM	(1927)
„ *orbiculata*	PALMER	(1928)
„ *totiseptata*	PALMER	(1928)
„ *vivari*	PALMER	(1928)

Chez *Sabinia sublacensis*, la cavité 3′ dans la valve supérieure, ne s'étend pas au delà du ligament du côté dorsal, tandisque la partie ventrale de cette cavité est plus grande. Il nous semble que ladite cavité s'agrandit du côté dorsal et se reduit du côté ventral progressivement pendant l'évolution phylogénique.

Les canaux polygonaux varient beaucoup, quant aux dimensions, en s'élargissant vers l'intérieur, tandisque chez notre espèce ils sont partout de la même largeur. La valve inférieure de sublacensis possède une cavité d'habitation beaucoup plus ample que celle de l'espèce nouvelle.

Il nous reste de mentionner que KLINGHARDT (l. c.) croit que *Sabinia sublacensis* n'est qu'une variation de *Sabina aniensis*, sans allégation de raisons.

L'espèce *aniensis* a une valve supérieure très courbée beaucoup plus courbée que celle de notre espèce. La section transversale est chez aniensis à peu près circulaire, chez la nouvelle espèce ovale angulaire avec une faible sillon près du ligament.

Surtout les cavités 3′ et C. V. (cavité viscerale = cavité d'habitation)

offrent de grandes différences: C. V. chez *aniensis* très irrégulière, chez la nouvelle espèce circulaire; 3′ d'*aniensis* pourvue d'une cavité accessoire énorme et ne dépassant le ligament du côté dorsal.

L'espèce *sinuata* a été éliminée par PARONA lui-même (KLINGHARDT, Die Rudisten I, 1921, pag. 55).

L'espèce *klinghardti* n'est pas du tout un *Sabinia*, mais un Sauvagésiné (fide M. MAC GILLAVRY).

Sabinia orbiculata a les valves-non seulement la valve supérieure, mais encore la valve inférieure-beaucoup plus courbées. Mais la cavité 3′ s'est développée au delà du ligament, comme chez *Sabinia kugleri*. Comme chez *Sabinia sublacensis* et *aniensis* les dimensions des canaux s'élargissent vers l'intérieur. PALMER n'a pas pu trouver la dent PIV dans ses sections.

L'espèce *totiseptata* n'a pas été décrite suffisamment. Dans la description de PALMER, on ne peut trouver qu'un seul caractère important. C'est la présence de canaux polygonaux cloisonnés, dont les uns (dans la valve supérieure) ont des cloisons planes, les autres (dans la valve inférieure) des cloisons concaves. Notre *Sabinia* a des cloisons planes dans les deux valves. PALMER dit que *totiseptata* n'a pas des canaux marginaux pyriformes. Si PALMER ne s'est pas trompé, ce serait un fait très remarquable. Notre espèce a en tous cas des canaux pyriformes dans les deux valves.

Sabinia vivari la troisième espèce de PALMER enfin n'a qu'une rainure ligamentaire externe très peu distincte. La cavité viscérale est encore plus petite que chez *Sabinia kugleri*. La courbure de la valve inférieure est plus étirée. PALMER ne mentionne pas des canaux marginaux pyriformes. Quoiqu'il en soit, nous n'avons pas des données suffisantes sur cette *Sabinia*.

Ici suit enfin une comparaison avec quelques Rudistides qui sont tellement semblables au genre *Sabinia* qu'on peut les confondre très facilement.

Comparaison avec le genre Coralliochama. Il y a deux différences principales: *Sabinia* a des canaux pyriformes dans les deux valves, *Coralliochama* en a dans la valve supérieure seulement, et puis le diamètre des canaux polygonaux chez *Coralliochama* se monte à environ 0,5 mm, chez *Sabinia* à 1 mm.

Comparaison avec Caprinuloidea multitubifera (PALMER 1928). Les dimensions des dents et des cavités de *Caprinuloidea multitubifera* et de *Sabinia kugleri* sont à peu près égales. Pourtant cette *Caprinuloidea* a beaucoup moins de canaux et ce qui constitue une différence très importante, les canaux marginaux pyriformes sont développés en deux ou trois séries plus ou moins distinctes.

Sabinia kugleri par contre n'en a qu'une seule série très régulière et distincte.

L'âge de Sabinia kugleri. Il n'y a que peu de certain à dire sur l'âge de la nouvelle espèce. Surtout parceque les fossiles ont été trouvés dans un bloc isolé de conglomérat.

En Amérique tous les espèces de *Sabinia* (décrites par PALMER) ont

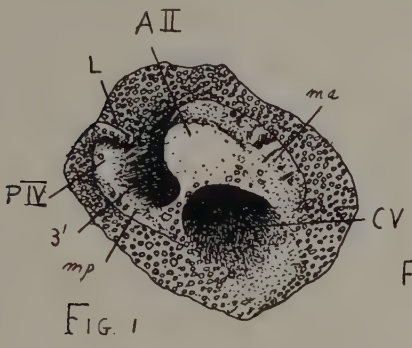

FIG. 1

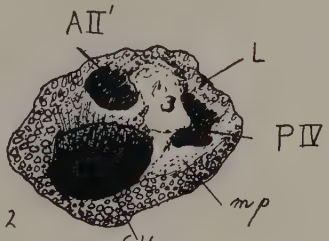

FIG. 2

FIG. 3

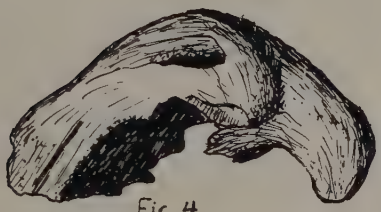
FIG. 4

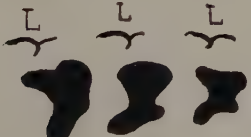

FIG. 5

FIG. 6

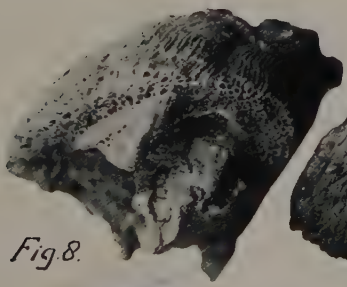
FIG. 8.

FIG. 7.

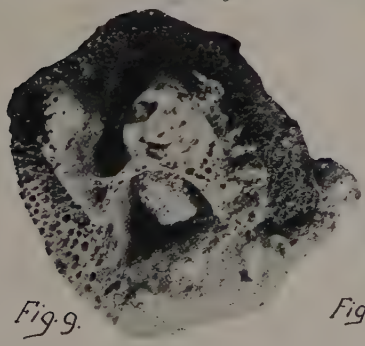
FIG. 9.

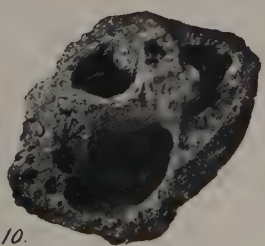
FIG. 10.

été trouvés dans des couches regardées par PALMER comme probablement cénomaniennes. En Italie les espèces de PARONA ont été trouvées dans des couches apparemment sénoniennes. La présence d'un foraminifère du genre *Gümbelina* (probablement *G. elegans*) pourrait fournir un argument en faveur d'un âge sénonien de *Sabinia kugleri*.

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EXPLICATIONS DES FIGURES.

Fig. 1: Valve supérieure vue du côté inférieure. × ½.

Fig. 2: Valve inférieure vue du côté supérieure. × ⅝.

Fig. 3: Valve inférieure. × ½.

Fig. 4: Deux valves inférieures adhérentes. × ¼.

Fig. 5: PIV′ chez trois valves inférieures différentes (fig. 5c PIV′ de la valve inférieure représentée dans la fig. 2). × ⅔.

Fig. 6: Série des canaux marginaux pyriformes dans la partie extérieure de la couche intérieure d'une valve inférieure. gr. nat.

Fig. 7: Valve supérieure. × ⅔.

Fig. 8: Valve supérieure vue obliquement. × ⅔.

Fig. 9: Valve supérieure vue comme dans la fig. 1. × ⅔.

Fig. 10: Valve inférieure vue comme dans la fig. 2. × ¾.

Comparative Pathology. — *The relation of head-length and head-index of* JOHANNSEN *and the spurious correlation of* PEARSON. By G. P. FRETS. (Communicated by Prof. J. BOEKE).

(Communicated at the meeting of April 24, 1937).

In a publication in 1907 JOHANNSEN discussed at length the fact that, if head measurements are arranged in groups according to the length, the average head-index decreases as the length increases. This observation was first made by him with beans.

The same fact was demonstrated by BOAS in 1899 with Indian material.

Similar calculations were made by me in 1922, following on JOHANNSEN and BOAS, and corresponding results obtained for the length and the head-index, also respectively for the breadth and the height and the head-index, for my own material and that of TOCHER.

In 1897 a communication was made in the Proceedings of the Royal Society by K. PEARSON, regarding what he calls "spurious correlation". He defines spurious correlation as "the correlation which will be found between indices, when the absolute values of the organs have been selected purely at random."

PEARSON also deduces a formula for this spurious correlation. Different statisticians have taken up this question. (YULE, GUMBEL).

Spurious correlation appears with the correlation of indices and, in general, if magnitudes of which the correlation is calculated are compounds and contain an equal component part; thus with indices, e.g., if one calculates the correlation of LB and LD, of LB and BD, of LD and BD; also with the correlation of L and LB, L and LD, but not with that of L and BD.

This spurious correlation is large. If spurious correlation appears, the correlation calculated can be taken as total or gross correlation and this is equal to the sum of the organic (true) correlation and the spurious correlation. Spurious correlation may be greater than the gross correlation.

In 1897 PEARSON stated "that the difficulty and danger which arise from the use of indices was brought home (to him) in an endeavour to deal with a considerable series of personal equation data" and that it was some time before he realised that "this spurious correlation had nothing to do with the manner of judging." He concludes that "the method which judges of the intensity of organic correlation by the reduction of all absolute measures to indices, is not free from obscurity."

YULE (1910) concurs with PEARSON in thinking that "the interpretation of correlation between indices is not free from obscurity", but in his view

"this obscurity is no less for correlations between absolute measurements." YULE then points out that we "at present have no real knowledge of the process of growth to guide us. We do not know the factors which determine the ultimate form of the individual."

With reference to PEARSON's publication, GALTON wrote a note, "believing that it might be useful in enabling others to realise the genesis of spurious correlation." (1897, p. 498). "The results arrived at," he says, "which are of serious interest, have at first sight a somewhat paradoxical appearance."[1])

Collaborators and pupils of PEARSON have determined the spurious correlation in the total correlation found by them. (WELDON, PEARL with the lower animals; FAWCETT, MACDONELL with the human skull).

FAWCETT (1902) found for Naqada skulls (Egypt) for 130 male skulls as correlation between L and $L\,B$ the gross correlation equals $-.551 \pm -.04$ and the spurious correlation $-.770 \pm .023$ (according to MACDONELL $-.694 \pm .031$); for 169 female skulls these figures are $-.560 \pm .037$ (acc. to MACDONELL $-.613 \pm .032$) and $-.624 \pm .033$ (acc. to M. $-.673 \pm .028$) respectively.

MACDONELL (1904) found for 131 English male crania the gross correlation of L and $L\,B$ equals $-.547 \pm .041$ and the spurious correlation $-.658 \pm .033$ and for 130 English female crania $-.541 \pm .042$ and $-.689 \pm .031$ respectively. In all these cases the spurious correlation is thus found to be greater than the gross correlation. As the organic correlation equals gross correlation-spurious correlation, the organic correlation is here positive.

JOHANNSEN does not mention the investigations of PEARSON regarding spurious correlation, nor PEARSON those of JOHANNSEN concerning the relation of head-length and index; PEARSON is acquainted with the publication of BOAS (1899).

In my investigations regarding the heredity of the dimensions and the indices of the seed of Phaseolus vulgaris, I found, the same as JOHANNSEN, that the average index decreases as the length of the beans increases (1934). I also noticed that, if the beans are grouped according to weight classes, there is a negative correlation of the length and the breadth for beans of the same weight class. Earlier (1922) I had found that there is a negative correlation of length and breadth for heads, of which $L + B + H$ (as expressing the size of the head) are the same. Like PEARSON and his pupils, I also found a negative correlation of L and $L\,B$.

In recent years (1932) TIMOFEEFF-RESSOVSKY and ZARAPKIN have determined with material of beetles that, besides dimensions and indices, the direction of the variations (das Gerichtetsein der Variationen) must be studied as well. As an example they also discuss the relation of the

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[1]) PEARSON (1924) defends spurious correlation against criticisms. "It seems to be the appropiate word to describe these correlations which are due, or largely due to "covering" factors" (p. 356).

skull-length and the skull-index. These authors make no communication from the literature.

If we endeavour to determine the significance of the findings of JOHANNSEN, that with increasing length the average index decreases, then it becomes clear that this relation includes spurious correlation.

We may replace this relation by the correlation of length and breadth for heads of the same capacity and then find that this correlation is negative, likewise by the correlation of the length and the index, which is also negative.

If in the correlation table of L and index we calculate the average index for each length class, we get the result obtained by JOHANNSEN, viz. with

TABLE 1. Classification of $L\,B$-indices according to head-length classes. 2765 measurements of the head. Twenty years and older. The material till 1929.

Length of the head	$L\,B$-Index					
	Males			Females		
	N	$M \pm m$	$\sigma \pm m$	N	$M \pm m$	$\sigma \pm m$
16.5				5	86.2	
17	2	80.8		29	84.42 ± 0.65	3.56 ± 0.47
17.5	8	82		197	83.1 ± 0.17	2.76 ± 0.14
18	62	82.41 ± 0.35	2.77 ± 0.25	483	81.29 ± 0.1	2.27 ± 0.07
18.5	150	81.65 ± 0.235	2.87 ± 0.165	506	80.28 ± 0.11	2.49 ± 0.08
19	318	80.05 ± 0.125	2.29 ± 0.09	296	79.1 ± 0.17	2.4 ± 0.1
19.5	317	79.17 ± 0.155	2.74 ± 0.115	80	78.3 ± 0.22	2 ± 0.16
20	199	78.39 ± 0.2	2.73 ± 0.165	17	77.4	
20.5	76	77.19 ± 0.3	2.58 ± 0.21	3	77.7	
21	17	75.74				
	1149			1616		

increasing length a decreasing average index. (FRETS, 1922, tab. 14—16, p. 512.)

The correlation of length and breadth of a head-material selected at random is positive. As stated above, I have already calculated before that for heads of equal size (expressed by $L + B + H$) and for beans of the same weight there exists a negative correlation of the length and the breadth.

In the following manner I have demonstrated the spurious correlation in my results.

PEARSON, when he regards the notion spurious correlation, says approximately: if an imp mixed up the observations belonging to each other and

if then the indices of these numbers not belonging to each other were calculated, and from these, two by two, the correlation, then one would obtain a correlation, and a rather high one.

I have done the same as PEARSON's imp, in order to demonstrate empirically the spurious correlation. I copied out the lengths and the breadths of the heads of adults of part of my material, arranged chronologically, and the indices and also the lengths, but now also showing the breadths in the reverse order in which they belong to the lengths; thus I now obtained side by side lengths and breadths, which do not belong to the same head, and from these dimensions calculated the "indices" [1]).

If, in accordance with JOHANNSEN, I calculate from these indices the average indices for the different length classes, then I also find here a decrease of the average indices with increasing lengths (tab. 2).

TABLE 2. Classification as in table 1 for "spurious heads", which are composed of the same material as of table 1.

Length of the head	*L B*-Index					
	Males			Females		
	N	$M \pm m$	$\sigma \pm m$	*N*	$M \pm m$	$\sigma \pm m$
16.5				6	89.2	
17	2	92 ± —		36	87.32 ± 0.5	3.03 ± 0.36
17.5	9	88.6 + —		187	84.27 ± 0.22	2.47 ± 0.16
18	63	85.3 ± 0.515	4.13 ± 0.365	478	82 ± 0.11	2.36 ± 0.1
18.5	150	83.27 ± 0.255	3.15 ± 0.18	505	80.1 ± 0.13	2.36 ± 0.9
19	315	80.83 ± 0.18	3.215 ± 0.13	294	78.12 ± 0.17	2.35 ± 0.12
19.5	322	78.9 ± 0.16	2.85 ± 0.11	82	76.9 ± 0.3	2.71 ± 0.22
20	199	76.65 ± 0.19	2.7 ± 0.135	18	74.2	
20.5	80	74.77 ± 0.71	3.21 ± 0.56	3	71.7	
21	17	74.1 ± —				
	1157			1609		

I likewise find a negative correlation of length and index, *L* and *L B*, (tab. 3).

A comparison of tables 1—3 shows that the spurious correlation is greater than the gross correlation. As gross correlation equals organic correlation plus spurious correlation, the organic correlation of *L* and *L B* is positive. (tab. 3).

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1) I see that WELDON determined the spurious correlation of his material in a corresponding manner.

TABLE 3. Correlation.

Correlation	Males		Females	
	N	$r \pm m$	N	$r \pm m$
Gross correlation of Land $L\,B$	1149	-0.426 ± 0.01	1616	-0.506 ± 0.02
Spurious „ „ „ „	1157	-0.698 ± 0.05	1609	-0.65 ± 0.014
Organic „ „ „ „		$+0.272$		$+0.144$
Stature and head-length (Material of TOCHER)	787	$+0.23 \pm 0.03$		

This is a remarkable result. JOHANNSEN (1907) does not discuss the significance of the relation found by him. He points out, it is true, that the length is a unit of measurement in the index and he mentions that for different bean species the results are not the same. He gives some examples in another field.

BOAS (1899) studies the biological significance of the index and, if he finds that with increasing head-length the average index falls, he points out that the cephalic index is (also) greatly influenced by causes other than the length and breadth of the head. No more than JOHANNSEN does BOAS recognise in his results the element of spurious correlation. He concludes (p. 460) "Among skulls belonging to the same type, a breath above the average is compensated by a height and length below the average. The law of compensation of VIRCHOW holds good also in normal skulls." "The correlation between length and breadth is not an expression of a biological relation between the two measurements, but an effect of the changes which both undergo when the capacity of the skull increases or decreases."

The negative correlation of head-length and head-breadth with skulls of the same capacity (or of length and breadth of beans of the same weight) was also explained by me (1922, 1934) as an expression of compensational growth in the sense of TSCHEPOURKOWSKY (1905). I would further mention that ORENSTEIN (1916) was struck by the "fact that the correlation coefficient for the length and the breadth of the head varies much from race to race" and he is inclined to believe that "a correlation really exists, but that its variation in value is very probably due to special factors, probably working independently and tending to modify the real measure of relationship. A close study of such factors is being undertaken" this author concluded.

TIMOFEEFF-RESSOVSKY and ZARAPKIN (1932a) put the problem as follows: "es kann die durch einen bestimmten Index charakterisierte Form in Abhängigkeit von der Grösze des betreffenden Merkmals variïren: das wird in den Fällen stattfinden, wenn die Raten der Gröszenzunahme der zwei in Index in Relation gebrachte Masze verschieden sind."

TIMOFEEFF constructs "Formtendenzkurven", e.g. for the skull material of MACDONELL. These authors conclude (1932b) regarding "das Gerichtetsein der Formvariation" "(dass) die Variationsrichtung (die Spezifizität der Manifestierung) vorwiegend, wenn nicht ausschliesslich erblich bedingt ist und gegenüber Milieueinflüszen sehr resistent."

It is clear that TIMOFEEFF overlooks the spurious correlation. An analysis of his results is also necessary and the share of the spurious correlation must be determined.

The correlation of L and I, i.e. $\frac{100\,B}{L}$ is negative; with increasing L, I becomes smaller, because in I, i.e. in $\frac{100\,B}{L}$, L appears in the denominator. This is the spurious correlation, which is an arithmetical phenomenon arising as a result of our comparing length and L B-index, and these possess the same factor.

Spurious correlation has nothing to do with compensational growth, as was assumed by BOAS (1899) and FRETS (1922) in explanation of the negative correlation found by them, nor with heredity, as TIMOFEEFF-RESSOVSKY assumes in explanation of the Gerichtetsein der Variabilität. Only when the spurious correlation is removed from the gross correlation found by these writers can the possibilities of explanation for the then remaining organic correlation be admitted.

As stated above, in so far as the correlation of L and L B differs from the spurious correlation, there is an organic correlation; thus organic correlation equals gross correlation minus spurious correlation. With increasing length the average index changes, because L appears in $\frac{100\,B}{L}$; B is a varying percentage of L.

Because the numbers ascend in arithmetical progression, the spurious correlation arises. This must be removed from the gross correlation found. In our case there remains then a small positive correlation.

These correlations are low and the results of others vary. (Cf. PEARSON and his pupils; PEARSON p. 495, FAWCETT p. 454, MACDONELL p. 244).

If the positive correlation has any significance, the explanation by compensational growth lapses[1]). A small positive correlation may be regarded as an expression of adaptability. From parents with very varying shapes and sizes of heads, a potentially very great length with a potentially very small breadth may be found together in the shape of the head of the children; it is possible that as an expression of adaptation the potentially

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[1]) D'ARCY W. THOMPSON in his book „Growth and Form" (1917) attaches importance to this principle. He considers the volume primary; "the volume is less affected than are the linear dimensions." (p. 776). "If thus a dimension changes, there will be an inclination with the other dimension to change in the opposite manner, in order to keep the volume unchanged." "The skull varies as a whole." These hypotheses are only to be proved empirically by further investigations.

very small breadth will be somewhat less small. Consequently for heads of the same group, so of equal length or of equal capacity, a small positive organic correlation of length and breadth would arise. (This small positive correlation of length and breadth is something else than the positive correlation of length and breadth of an arbitrary head-material, so of heads of different capacity; it is the difference of "interracial" and "intraracial" correlation). Still further investigation is necessary here with large and reliable material. PEARL also, on the lines of DRIESCH, takes up the general significance of the small positive correlation found by him in his material. He argues that DRIESCH's law of the proportionality of the parts, that is independent of the size, does not hold. There is a significant correlation between shape of body and absolute size.

Finally an observation regarding the negative correlation of stature and *L B*-index may be made. This negative correlation also, in my opinion, contains spurious correlation. The head-length is partly an extension of the body axis, i.e. a component part of the stature; there is a positive correlation of stature and head-length (tab. 3). The negative correlation of stature and *L B*-index is thus partly the same as that of head-length and *L B*-index and consequently contains spurious correlation. (Vide also JOHANNSEN).

In this communication I have aimed at showing that different authors, partly in other terms, have discussed the correlation of head-length and *L B*-index and that this correlation contains an untrue or spurious correlation not belonging to the organic correlation, which spurious correlation is fairly great, in our case greater than the total correlation.

It is necessary that this spurious correlation be taken into consideration in anthropology. Besides the index, also the dimensions of the material investigated must, therefore, always be mentioned.

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